

MICROWAVE ENGINEERING

LECTURE NOTES

**B.TECH
(IV YEAR – I SEM)
(2019-2020)**

Prepared by:

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MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY
(Autonomous Institution – UGC, Govt. of India)

Recognized under 2(f) and 12 (B) of UGC ACT 1956

(Affiliated to JNTUH, Hyderabad, Approved by AICTE - Accredited by NBA & NAAC – 'A' Grade - ISO 9001:2015 Certified)
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(R15A0421) MICROWAVE ENGINEERING OBJECTIVES

1. To analyze micro-wave circuits incorporating hollow, dielectric and planar waveguides, transmission lines, filters and other passive components, active devices.
2. To Use S-parameter terminology to describe circuits.
3. To explain how microwave devices and circuits are characterized in terms of their “S” Parameters.
4. To give students an understanding of microwave transmission lines.
5. To Use microwave components such as isolators, Couplers, Circulators, Tees, Gytrators etc..
6. To give students an understanding of basic microwave devices (both amplifiers and oscillators).
7. To expose the students to the basic methods of microwave measurements.

UNIT I:

Waveguides & Resonators: Introduction, Microwave spectrum and bands, applications of Microwaves, Rectangular Waveguides-Solution of Wave Equation in Rectangular Coordinates, TE/TM mode analysis, Expressions for fields, Cutoff frequencies, filter characteristics, dominant and degenerate modes, sketches of TE and TM mode fields in the cross-section, Mode characteristics - Phase and Group velocities, wavelengths and impedance relations, Rectangular Waveguides – Power Transmission and Power Losses, Impossibility of TEM Modes, Micro strip Lines-Introduction, Z₀ Relations, losses, Q-factor, Cavity resonators-introduction, Rectangular and cylindrical cavities, dominant modes and resonant frequencies, Q-factor and coupling coefficients, Illustrative Problems.

UNIT II:

Waveguide Components-I: Scattering Matrix - Significance, Formulation and properties, Wave guide multiport junctions - E plane and H plane Tees, Magic Tee, 2-hole Directional coupler, S Matrix calculations for E plane and H plane Tees, Magic Tee, Directional coupler, Coupling mechanisms - Probe, Loop, Aperture types, Wave guide discontinuities - Waveguide Windows, tuning screws and posts, Irises, Transitions, Twists, Bends, Corners and matched loads, Illustrative Problems.

Waveguide Components-II: Ferrites composition and characteristics, Faraday rotation, Ferrite components - Gytrator, Isolator, Circulator.

UNIT III:

Linear beam Tubes: Limitations and losses of conventional tubes at microwave frequencies, Classification of Microwave tubes, **O type tubes** - 2 cavity klystrons-structure, Reentrant cavities, velocity modulation process and Applegate diagram, bunching process and small signal theory Expressions for o/p power and efficiency, Reflex Klystrons-structure, Velocity Modulation, Applegate diagram, mathematical theory of bunching, power output, efficiency, oscillating modes and o/p characteristics, Effect of Repeller Voltage on Power o/p, Significance, types and characteristics of slow wave structures, structure of TWT and amplification process (qualitative treatment), Suppression of oscillations, Gain considerations.

UNIT IV:

Cross-field Tubes: Introduction, Cross field effects, Magnetrons- different types, cylindrical travelling wave magnetron-Hull cutoff and Hartree conditions, modes of resonance and PI-mode operation, separation of PI-mode, O/P characteristics.

Microwave Semiconductor Devices: Introduction to Microwave semiconductor devices, classification, applications, Transfer Electronic Devices, Gunn diode - principles, RWH theory, Characteristics, Basic modes of operation - Gunn oscillation modes, LSA Mode, Introduction to Avalanche Transit time devices (brief treatment only), Illustrative Problems.

UNIT V:

Microwave Measurements: Description of Microwave Bench – Different Blocks and their Features, Precautions; Waveguide Attenuators – Resistive Card, Rotary Vane types; Waveguide Phase Shifters – Dielectric, Rotary Vane types. Microwave Power Measurement – Bolometer Method. Measurement of Attenuation, Frequency, VSWR, Cavity Q. Impedance Measurements.

TEXT BOOKS:

8. Microwave Devices and Circuits – Samuel Y. Liao, PHI, 3rd Edition, 1994.
9. Microwave and Radar Engineering- M.Kulkarni, Umesh Publications, 1998.

REFERENCES :

1. Foundations for Microwave Engineering – R.E. Collin, IEEE Press, John Wiley, 2nd Edition, 2002.
10. Microwave Circuits and Passive Devices – M.L. Sisodia and G.S.Raghuvanshi, Wiley Eastern Ltd., New Age International Publishers Ltd., 1995.
11. Microwave Engineering Passive Circuits – Peter A. Rizzi, PHI, 1999.
12. Electronic and Radio Engineering – F.E. Terman, McGraw-Hill, 4th ed., 1955.
13. Elements of Microwave Engineering – R. Chatterjee, Affiliated East-West Press Pvt. Ltd., New Delhi, 1988.

OUTCOMES

14. Understand the significance of microwaves and microwave transmission lines
15. Analyze the characteristics of microwave tubes and compare them
16. Be able to list and explain the various microwave solid state devices
17. Can set up a microwave bench for measuring microwave parameters

UNIT- I

MICROWAVE TRANSMISSION LINES-I

INTRODUCITON

Microwaves are electromagnetic waves with frequencies between 300MHz (0.3GHz) and 300GHz in the electromagnetic spectrum.

Radio waves are electromagnetic waves within the frequencies 30KHz - 300GHz, and include microwaves. Microwaves are at the higher frequency end of the radio wave band and low frequency radio waves are at the lower frequency end.

Mobile phones, phone mast antennas (base stations), DECT cordless phones, Wi-Fi, WLAN, WiMAX and Bluetooth have carrier wave frequencies within the microwave band of the electromagnetic spectrum, and are pulsed/modulated. Most Wi-Fi computers in schools use 2.45GHz (carrier wave), the same frequency as microwave ovens. Information about the frequencies can be found in Wi-Fi exposures and guidelines.

It is worth noting that the electromagnetic spectrum is divided into different bands based on frequency. But the biological effects of electromagnetic radiation do not necessarily fit into these artificial divisions.

A waveguide consists of a hollow metallic tube of either rectangular or circular cross section used to guide electromagnetic wave. Rectangular waveguide is most commonly used as waveguide. waveguides are used at frequencies in the microwave range.

At microwave frequencies (above 1GHz to 100 GHz) the losses in the two line transmission system will be very high and hence it cannot be used at those frequencies . hence microwave signals are propagated through the waveguides in order to minimize the losses.

Properties and characteristics of waveguide:

1. The conducting walls of the guide confine the electromagnetic fields and thereby guide the electromagnetic wave through multiple reflections .
2. when the waves travel longitudinally down the guide, the plane waves are reflected from wall to wall .the process results in a component of either electric or

magnetic fields in the direction of propagation of the resultant wave.

3. TEM waves cannot propagate through the waveguide since it requires an axial conductor for axial current flow .
4. when the wavelength inside the waveguide differs from that outside the guide, the velocity of wave propagation inside the waveguide must also be different from that through free space.
5. if one end of the waveguide is closed using a shorting plate and allowed a wave to propagate from other end, then there will be complete reflection of the waves resulting in standing waves.

Waveguides

A waveguide consists of a hollow metallic tube of a rectangular or circular shape used to guide an electromagnetic wave.

Waveguides are used principally at frequencies in the microwave range.

In waveguide the electric and magnetic fields are confined the space with in the guides. Thus no power is lost through radiation and even the dielectric loss is negligible since the guides are normally air-filled. However, there is some power loss as heat in the walls of the guide, but the loss is very small.

It is possible to propagate several modes of EM waves with in a waveguide. These modes correspond to solutions of Maxwell's Equations for particular waveguide.

If the frequency of the impressed signal is above the cut-off frequency for a given mode, the EM energy can be transmitted through the guide for that particular mode without attenuation.

The mode which is having the lowest cut-off frequency is called the 'Dominant Mode'

Waveguide are two types

i) Rectangular waveguide ii) Circular waveguide

Rectangular Waveguide

A Rectangular waveguide is a hollow metallic tube with a rectangular cross section.

When the waves travel longitudinally down the guide because of conducting walls plane waves are reflected from wall to

wall. This process results in a component of either electric or magnetic field in the direction of propagation of the resultant wave. Therefore the wave is no longer a transverse electromagnetic wave.

Any uniform plane wave in a lossless guide may be resolved into TE and TM waves.

In rectangular guide the modes are designed TE_{mn} or TM_{mn} .

Propagation of waves in Rectangular waveguides

Consider a rectangular waveguide situated in the rectangular coordinate system with its breadth along x-axis, width along y-axis and the wave is assume to propagate along the z-direction. Waveguide is filled with air. In a waveguide no TEM wave is exists.

TEM(Transverse Electromagnetic wave): in TEM both electric and magnetic fields are purely transverse to the direction of propagation and consequens have no 'z' directed E & H components.

TE(Transverse Electric Wave) In TE wave only the E field is purely transverse to the direction of propagation and the magnetic field is not purely transverse

i.e. $E_z=0, H_z \neq 0$

TM(Transverse Magnetic Wave) In TE wave only the H field is purely transverse to the direction of propagation and the Electric field is not purely transverse

i.e. $E_z \neq 0, H_z=0$

HE(Hybrid wave) In this neither electric nor magnetic fields are purely transverse to the direction of propagation.

i.e. $E_z \neq 0, H_z \neq 0$

WAVE EQUATIONS

Since we assumed that the wave direction is along z-direction then the wave equation are

$$\nabla^2 E_z = -\omega^2 \mu \epsilon E_z \quad \text{for TM wave} \text{-----(1)}$$

$$\nabla^2 H_z = -\omega^2 \mu \epsilon H_z \quad \text{for TE wave} \text{-----(2)}$$

$$\text{Where } E_z = E_0 e^{-\gamma z}, H_z = H_0 e^{-\gamma z} \text{-----(3)}$$

The condition for wave propagation is that γ must be imaginary.

Differentiating eqn(3) w.r.t 'z' we get

$$\partial E_z / \partial z = E_0 e^{-\gamma z} (-\gamma) = -\gamma E_z \text{-----(4)}$$

$$\text{Hence we can define operator } \partial / \partial z = -\gamma \text{-----(5)}$$

By differentiating eqn(4) w.r.t 'z' we get

$$\partial^2 E_z / \partial z^2 = \gamma^2 E_z$$

We can define the operator

$$\partial^2 / \partial z^2 = \gamma^2 \text{-----(6)}$$

From eqn(1) we can write

$$\nabla^2 E_z = -\omega^2 \mu \epsilon E_z$$

By expanding $\nabla^2 E_z$ in rectangular coordinate system

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0 \text{ for TM wave-----(7)}$$

$$\text{Similarly} \qquad \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0 \text{ for TE wave-----(8)}$$

By solving above two partial differential equations we get solutions for E_z and H_z . Using Maxwell's equations. it is possible to find the various components along x and y-directions.

From Maxwell's first equation, we have

$$\nabla \times H = j\omega \epsilon E$$

$$\begin{matrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{matrix} = j\omega \epsilon [E_x a_x + E_y a_y + E_z a_z]$$

$$a_x \rightarrow \gamma H_y + \partial H_z / \partial y = j\omega \epsilon E_x \text{-----(9)}$$

$$a_y \rightarrow \gamma H_x + \partial H_z / \partial x = -j\omega \epsilon E_y \text{-----(10)}$$

$$a_z \rightarrow \partial H_y / \partial x + \partial H_x / \partial y = j\omega \epsilon E_z \text{-----(11)}$$

similarly from Maxwell's 2nd equation we have

$$\nabla \times E = -j\omega \mu H$$

By expanding

$$\begin{matrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{matrix} = -j\omega [\mu H_x a_x + \mu H_y a_y + \mu H_z a_z]$$

Since $\partial/\partial z = \gamma$

$$\begin{matrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & -\gamma \\ E_x & E_y & E_z \end{matrix} = -j\omega [\mu H_x a_x + \mu H_y a_y + \mu H_z a_z]$$

By comparing a_x, a_y, a_z components

$$a_x \rightarrow \gamma E_y + \partial E_z / \partial y = -j\omega \mu H_x \text{-----(12)}$$

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y \text{-----(13)}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\epsilon H_z \text{-----(14)}$$

From eqn(13)

$$H_y = \left[\gamma E_x + \frac{\partial E_z}{\partial x} \right] / j\omega\mu \text{-----(15)}$$

By substituting eqn(15) in eqn(9) we get

$$\gamma^2 / j\omega\mu E_x + \gamma / j\omega\epsilon \frac{\partial E_z}{\partial x} + \frac{\partial H_z}{\partial y} = j\omega\epsilon E_x$$

$$\text{since } \gamma^2 + \omega^2\mu\epsilon = h^2$$

by dividing the above equation with h^2 we get

$$E_x = -\gamma/h^2 \frac{\partial E_z}{\partial x} - j\omega\mu/h^2 \frac{\partial H_z}{\partial y} \text{-----(15)}$$

Similarly

$$E_y = -\gamma/h^2 \frac{\partial E_z}{\partial x} + j\omega\epsilon/h^2 \frac{\partial E_z}{\partial y} \text{-----(16)}$$

And

$$H_x = -\gamma/h^2 \frac{\partial H_z}{\partial x} + j\omega\mu/h^2 \frac{\partial E_z}{\partial y} \text{-----(17)}$$

$$H_y = -\gamma/h^2 \frac{\partial H_z}{\partial y} - j\omega\mu/h^2 \frac{\partial E_z}{\partial x} \text{-----(18)}$$

These equations give a general relationship for field components with in a waveguide.

Propagation of TEM Waves:

For TEM wave

$$E_z = 0 \text{ and } H_z = 0$$

Substituting these values in equns (15) to (18) all the field components along x and y directions E_x, E_y, H_x, H_y vanish and have a TEM wave cannot exist inside a waveguide.

Modes

The electromagnetic wave inside a waveguide can have an infinite number of patterns which are called modes.

The electric field cannot have a component parallel to the surface i.e. the electric field must always be perpendicular to the surface at the conductor.

The magnetic field on the other hand always parallel to the surface of the conductor and cannot have a component perpendicular to it at the surface.

TE Mode Analysis

The TE_{mn} modes in a rectangular waveguide are characterized by E_z=0. The z component of the magnetic field, H_z must exist in order to have energy transmission in the guide.

The wave equation for TE wave is given by

$$\nabla^2 H_z = -\omega^2 \mu \epsilon H_z \text{-----(1)}$$

$$\text{i.e. } \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = -\omega^2 \mu \epsilon H_z$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z + \omega^2 \mu \epsilon H_z = 0$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu \epsilon) H_z = 0 \quad \gamma^2 + \omega^2 \mu \epsilon = h^2$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0 \text{-----(2)}$$

This is a partial differential equation whose solution can be assumed.

Assume a solution

$$H_z = XY$$

Where X=pure function of x only

Y= pure function of y only

From equation 2

$$\frac{\partial^2 [XY]}{\partial x^2} + \frac{\partial^2 [XY]}{\partial y^2} + h^2 XY = 0$$

$$Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} + h^2 XY = 0$$

Dividing above equation with XY on both sides

$$1/X \frac{\partial^2 X}{\partial x^2} + 1/Y \frac{\partial^2 Y}{\partial y^2} + h^2 = 0 \text{-----(3)}$$

Here $1/X \frac{\partial^2 X}{\partial x^2}$ is purely a function of x and $1/Y \frac{\partial^2 Y}{\partial y^2}$ is purely a function of y

$$\text{Let } 1/X \frac{\partial^2 X}{\partial x^2} = -B^2 \text{ \& } 1/Y \frac{\partial^2 Y}{\partial y^2} = -A^2$$

i.e. from equation (3)

$$-B^2 - A^2 + h^2 = 0$$

$$\text{i.e. } h^2 = A^2 + B^2 \text{-----(4)}$$

$$X = c_1 \cos Bx + c_2 \sin Bx$$

$$Y = c_3 \cos Ay + c_4 \sin Ay$$

i.e. the complete solution for H_z=XY is

$$H_z = (c_1 \cos Bx + c_2 \sin Bx)(c_3 \cos Ay + c_4 \sin Ay) \text{-----(5)}$$

Where c_1, c_2, c_3 and c_4 are constants which can be evaluated by applying boundary conditions.

Boundary Conditions

Since we consider a TE wave propagating along z direction. So $E_z=0$ but we have components along x and y direction.

$E_x=0$ waves along bottom and top walls of the waveguide

$E_y=0$ waves along left and right walls of the waveguide

1st Boundary condition:

$E_x=0$ at $y=0 \forall x \rightarrow 0 \text{ to } a$ (bottom wall)

2nd Boundary condition

$E_x=0$ at $y=b \forall x \rightarrow 0 \text{ to } a$ (top wall)

3rd Boundary condition

$E_y=0$ at $x=0 \forall y \rightarrow 0 \text{ to } b$ (left side wall)

4th Boundary condition

$E_y=0$ at $x=a \forall y \rightarrow 0 \text{ to } b$ (right side wall)

i) Substituting 1st Boundary condition in eqn(5)

Since we have

$$E_x = -\gamma/h^2 \partial E_z / \partial x - j\omega\mu/h^2 \partial H_z / \partial y \text{-----(6)}$$

Since $E_z=0 \rightarrow E_x = -j\omega\mu/h^2 \partial [(c_1 \cos Bx + c_2 \sin Bx)(c_3 \cos Ay + c_4 \sin Ay)] / \partial y$

$$E_x = -j\omega\mu/h^2 \partial [(c_1 \cos Bx + c_2 \sin Bx)(-A c_3 \sin Ay + A c_4 \cos Ay)] / \partial y$$

From the first boundary condition we get

$$0 = -j\omega\mu/h^2 \partial [(c_1 \cos Bx + c_2 \sin Bx) \neq 0, A \neq 0]$$

$$c_4 = 0$$

Substituting the value of c_4 in eqn (5), the solution reduces to

$$H_z = (c_1 \cos Bx + c_2 \sin Bx)(c_3 \cos Ay) \text{-----(7)}$$

ii) from third boundary condition

$E_y=0$ at $x=0 \forall y \rightarrow 0 \text{ to } b$

Since we have

$$E_y = -\gamma/h^2 \partial E_z / \partial y + j\omega\mu/h^2 \partial H_z / \partial x \text{-----(8)}$$

Since $E_z=0$ and substituting the value of H_z in eqn(7), we get

$$E_y = j\omega\mu/h^2 \partial [(c_1 \cos Bx + c_2 \sin Bx)(c_3 \cos Ay)] / \partial x$$

$$E_y = j\omega\mu/h^2 [(-B c_1 \sin Bx + B c_2 \sin Bx)(c_3 \cos Ay)]$$

From third condition,

$$0 = j\omega\mu/h^2(0 + Bc^2)c_3\cos Ay$$

Since $\cos Ay \neq 0, B \neq 0, c_3 \neq 0$

$$c_2 = 0$$

from eq (7)

$$H_z = c_1 c_3 \cos Bx \cos Ay \text{-----(9)}$$

iii) 2nd Boundary condition

since we have

$$E_x = -\gamma/h^2 \partial E_z / \partial x - j\omega\mu/h^2 \partial H_z / \partial y$$

$$= -j\omega\mu/h^2 \partial / \partial y [c_1 c_3 \cos Bx \cos Ay] [E_z = 0]$$

$$E_x = j\omega\mu/h^2 c_1 c_3 \cos Bx \sin Ay$$

From the second boundary condition,

$$E_x = 0 \text{ at } y = b \quad \forall x \rightarrow 0 \text{ to } a$$

$$0 = j\omega\mu/h^2 c_1 c_3 \cos Bx \sin Ab$$

$$\cos Bx \neq 0, c_1 c_3 \neq 0$$

$$\sin Ab = 0 \text{ or } Ab = n\pi \text{ where } n = 0, 1, 2, \dots$$

$$A = n\pi/b \text{-----(10)}$$

iv) 4th Boundary condition

since

$$E_y = -\gamma/h^2 \partial E_z / \partial y + j\omega\mu/h^2 \partial H_z / \partial x$$

$$E_y = -j\omega\mu/h^2 \partial / \partial x [c_1 c_3 \cos Bx \cos Ay]$$

$$E_y = -j\omega\mu/h^2 c_1 c_3 \sin Bx \cdot B \cos Ay$$

From the 4th Boundary condition

$$E_y = 0 \text{ at } x = a \quad \forall y \rightarrow 0 \text{ to } b$$

$$0 = -j\omega\mu/h^2 B c_1 c_3 \sin Bx \cdot \cos Ay \quad \forall y \rightarrow 0 \text{ to } b$$

$$\cos Ay \neq 0, c_1 c_3 \neq 0$$

$$\sin Ba = 0$$

$$B = m\pi/a \text{-----(11)}$$

From eq(9)

$$H_z = c_1 c_3 \cos(m\pi/a) x \cos(n\pi/b) y$$

Let $c_1 c_3 = c$

$$H_z = c \cos(m\pi/a) x \cos(n\pi/b) y e^{(j\omega t - \gamma z)} \text{-----(12)}$$

Field Components

$$E_x = -\gamma/h^2 \partial E_z / \partial x - j\omega\mu/h^2 \partial H_z / \partial y$$

Since $E_z = 0$ for TE wave

$$E_x = j\omega\mu/h^2 c(n\pi/b) \cos(m\pi/a) x \sin(n\pi/b) y e^{(j\omega t - \gamma z)} \text{-----(13)}$$

$$E_y = -\gamma/h^2 \partial E_z / \partial y + j\omega\mu/h^2 \partial H_z / \partial x$$

Since $E_z = 0$ for TE wave

$$E_y = j\omega\mu/h^2 \partial H_z / \partial x$$

$$E_y = -j\omega\mu/h^2 c[m\pi/a] \sin(m\pi/a) x \cos(n\pi/b) y e^{(j\omega t - \gamma z)} \text{-----(14)}$$

Similarly

$$H_x = -\gamma/h^2 \partial H_z / \partial x - j\omega\epsilon/h^2 \partial E_z / \partial y$$

$$H_x = \gamma/h^2 c(m\pi/a) \sin(m\pi/a) x \cos(n\pi/b) y e^{(j\omega t - \gamma z)} \text{-----(15)}$$

$$H_y = -\gamma/h^2 \partial H_z / \partial y - j\omega\epsilon/h^2 \partial E_z / \partial x$$

$$H_y = -\gamma/h^2 c(n\pi/b)^2 \cos(m\pi/a) x \sin(n\pi/b) y e^{j\omega t - \gamma z} \text{-----(16)}$$

TM Mode Analysis

For TM wave $H_z = 0$ $E_z \neq 0$

$$\partial^2 E_z / \partial x^2 + \partial^2 E_z / \partial y^2 + h^2 E_z = 0 \text{-----(1)}$$

This is a partial differential equation which can be solved to get the different field components E_x, E_y, H_x and H_y by variable separable method.

Let us assume a solution

$$E_z = XY \text{-----(2)}$$

Using these two equations from eqn(1) we get

$$Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} + h^2 XY = 0 \text{-----(3)}$$

Dividing above equation with XY on both sides

$$1/X \frac{\partial^2 X}{\partial x^2} + 1/Y \frac{\partial^2 Y}{\partial y^2} + h^2 = 0 \text{-----(4)}$$

Here $1/X \frac{\partial^2 X}{\partial x^2}$ is purely a function of x and $1/Y \frac{\partial^2 Y}{\partial y^2}$ is purely a function of y

$$\text{Let } 1/X \frac{\partial^2 X}{\partial x^2} = -B^2 \text{-----(5)}$$

$$1/Y \frac{\partial^2 Y}{\partial y^2} = -A^2 \text{-----(6)}$$

i.e. from equation (4),(5) and(6)

$$-B^2 - A^2 + h^2 = 0$$

$$\text{i.e. } h^2 = A^2 + B^2 \text{-----(7)}$$

the solution of eqn(5) and(6) are

$$X = c_1 \cos Bx + c_2 \sin Bx$$

$$Y = c_3 \cos Ay + c_4 \sin Ay$$

Where c_1, c_2, c_3 and c_4 are constants which can be evaluated by applying boundary conditions

From eqn(1)

$$E_z = XY$$

$$E_z = (c_1 \cos Bx + c_2 \sin Bx)(c_3 \cos Ay + c_4 \sin Ay) \text{----(10)}$$

Boundary Conditions

Since we consider a TE wave propagating along z direction. So $E_z = 0$ but we have components along x and y direction.

$E_x = 0$ waves along bottom and top walls of the waveguide

$E_y = 0$ waves along left and right walls of the waveguide

1st Boundary condition:

$$E_x = 0 \text{ at } y=0 \forall x \rightarrow 0 \text{ to } a \text{ (bottom wall)}$$

2nd Boundary condition

$$E_x = 0 \text{ at } y=b \forall x \rightarrow 0 \text{ to } a \text{ (top wall)}$$

3rd Boundary condition

$$E_y = 0 \text{ at } x=0 \forall y \rightarrow 0 \text{ to } b \text{ (left side wall)}$$

4th Boundary condition

$$E_y = 0 \text{ at } x=a \forall y \rightarrow 0 \text{ to } b \text{ (right side wall)}$$

i) Substituting 1st Boundary condition in eqn(10)

Since we have

$$0 = E_z = [c_1 \cos Bx + c_2 \sin Bx][c_3 \cos A0 + c_4 \sin A0]$$

$$[c_1 \cos Bx + c_2 \sin Bx]c_3 = 0$$

$$c_1 \cos Bx + c_2 \sin Bx \neq 0$$

$$c_3 = 0$$

$$\text{i.e. } E_z = [c_1 \cos Bx + c_2 \sin Bx]c_4 \sin Ay \text{-----(11)}$$

ii) Substituting 2nd Boundary condition in eqn(11), we get

$$E_z = c_2 c_4 \sin B x \sin A y \text{-----(12)}$$

iii) Substituting 3rd Boundary condition in eqn(12), we get

$$\sin A b = 0$$

$$A = n\pi/b \text{-----(13)}$$

iv) Substituting 4th Boundary condition in eqn(12), we get

$$\sin B a = 0$$

$$B = m\pi/a \text{-----(14)}$$

From (12),(13),(14)

$$E_z = c \sin(m\pi/a) x \sin(n\pi/b) y e^{j(\omega t - \gamma z)} \text{-----(15)}$$

$$E_x = -\gamma/h^2 \partial E_z / \partial x$$

$$E_x = -\gamma/h^2 c (m\pi/a) \cos(m\pi/a) x \sin(n\pi/b) y e^{j\omega t - \gamma z} \text{-----(16)}$$

$$E_y = -\gamma/h^2 c (n\pi/b) \sin(m\pi/a) x \cos(n\pi/b) y e^{j\omega t - \gamma z} \text{-----(17)}$$

$$H_x = j\omega\epsilon/h^2 c (n\pi/b) \sin(m\pi/a) x \cos(n\pi/b) y e^{j\omega t - \gamma z} \text{-----(18)}$$

$$H_y = j\omega\epsilon/h^2 c [m\pi/a] \cos(m\pi/a) x \sin(n\pi/b) y e^{j\omega t - \gamma z} \text{-----(19)}$$

Cut-off Frequency of a Waveguide

Since we have

$$\gamma^2 + \omega^2 \mu\epsilon = h^2 = A^2 + B^2$$

$$A = n\pi/b, B = m\pi/a$$

$$\gamma^2 = (m\pi/a)^2 + (n\pi/b)^2 - \omega^2 \mu\epsilon$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + (n\pi/b)^2 - \omega^2 \mu\epsilon} = \alpha + j\beta$$

At lower frequencies

$$\gamma > 0$$

$$\sqrt{\left(\frac{m\pi}{a}\right)^2 + (n\pi/b)^2 - \omega^2 \mu\epsilon} > 0$$

γ then becomes real and positive and equal to the attenuation constant α i.e. the wave is completely attenuated and there is no phase change. Hence the wave cannot propagate.

However at higher frequencies, $\gamma < 0$

$$\sqrt{\left(\frac{m\pi}{a}\right)^2 + (n\pi/b)^2 - \omega^2 \mu\epsilon} < 0$$

γ becomes imaginary there will be phase change β and hence the wave propagates.

At the transition γ becomes zero and the propagation starts. The frequency at which γ just becomes zero is defined as the cut-off frequency f_c

At $f=f_c, \gamma=0$

$$0=(m\pi/a)^2+(n\pi/b)^2-\omega_c^2\mu\epsilon \text{ or}$$

$$f_c=1/2\pi\sqrt{\mu\epsilon}[(m\pi/a)^2+(n\pi/b)^2]^{1/2}$$

$$f_c=c/2[(m\pi/a)^2+(n\pi/b)^2]^{1/2}$$

The cut-off wavelength(λ_c) is

$$\lambda_c=c/f_c=c/2[(m\pi/a)^2+(n\pi/b)^2]^{1/2}$$

$$\lambda_{cm,n}=2ab/[m^2b^2+n^2a^2]^{1/2}$$

All wavelengths greater than λ_c are attenuated and these less than λ_c are allowed to propagate inside the waveguide.

Guided Wavelength (λ_g)

It is defined as the distance travelled by the wave in order to undergo a phase shift of 2π radians.

It is related to phase constant by the relation

$$\lambda_g=2\pi/\beta$$

the wavelength in the waveguide is different from the wavelength in free space. Guide wavelength is related to free space wavelength λ_0 and cut-off wavelength λ_c by

$$1/\lambda_g^2=1/\lambda_0^2-1/\lambda_c^2$$

The above equation is true for any mode in a waveguide of any cross section

Phase Velocity(v_p)

Wave propagates in the waveguide when guide wavelength λ_g is greater than the free space wavelength λ_0 .

In a waveguide, $v_p = \lambda_g f$ where v_p is the phase velocity. But the speed of light is equal to product of λ_0 and f . This v_p is greater than the speed of light since $\lambda_g > \lambda_0$.

The wavelength in the guide is the length of the cycle and v_p represents the velocity of the phase.

It is defined as the rate at which the wave changes its phase in terms of the guide wavelength.

$$V_p=\omega/\beta$$

$$V_p=c/[1-(\lambda_0/\lambda_c)^2]^{1/2}$$

Group Velocity(v_g)

The rate at which the wave propagates through the waveguide and is given by

$$V_g=d\omega/d\beta$$

Since $\beta = [\mu\epsilon(\omega^2 - \omega_c^2)]^{1/2}$

Now differentiating β w.r.t ω we get

$$V_g = c[1 - (\lambda_0/\lambda_c)^2]^{1/2}$$

Consider the product of V_p and V_g

$$V_p \cdot V_g = c^2$$

Dominant Mode

The mode for which the cut-off wavelength assumes a maximum value.

$$\lambda_{cmn} = \frac{2ab}{\sqrt{m^2b^2 + n^2a^2}}$$

Dominant mode in TE

For TE_{01} mode $\lambda_{c01} = 2b$

TE_{10} mode $\lambda_{c10} = 2a$

Among all λ_{c10} has the maximum value since 'a' is the larger dimensions than 'b'. Hence TE_{10} mode is the dominant mode in rectangular waveguide.

Dominant Mode in TM

Minimum possible mode is TM_{11} . Higher modes than this also exist.

Degenerate Modes

Two or more modes having the same cut-off frequency are called 'Degenerate modes'

For a rectangular waveguide TE_{mn}/TM_{mn} modes for which both $m \neq 0, n \neq 0$ will always be degenerate modes.

Wavelengths and Impedance Relations[TE & TM WAVES]

Guide Wavelength(λ_g)

It is defined as the distance travelled by the wave in order to undergo a phase shift of 2π radians.

$$1/\lambda_g^2 = 1/\lambda_0^2 - 1/\lambda_c^2$$

Wave impedance is defined as the ratio of the strength of electric field in one transverse direction to the strength of the magnetic field along the other transverse direction.

$$Z_Z = E_x/H_y$$

1) Wave impedance for a TM wave in rectangular waveguide

$$Z_Z = -\gamma/h^2 \partial E_z / \partial x - j\omega\mu \partial H_z / \partial y / -\gamma/h^2 \partial H_z / \partial y - j\omega\epsilon/h^2 \partial E_z / \partial x$$

For a TM wave $H_z=0$

$$Z_{TM} = \gamma / j\omega\epsilon$$

$$= \beta / \omega\epsilon$$

Since we have $\beta = [\omega^2\mu\epsilon - \omega_c^2\mu\epsilon]^{1/2}$

$$Z_{TM} = \eta [1 - (\lambda_0/\lambda_c)^2]^{1/2}$$

Since λ_0 is always less than λ_c for wave propagation $Z_{TM} < \eta$

2) Wave impedance of TE waves in rectangular waveguide

$$Z_{TE} = \eta / [1 - (\lambda_0/\lambda_c)^2]^{1/2}$$

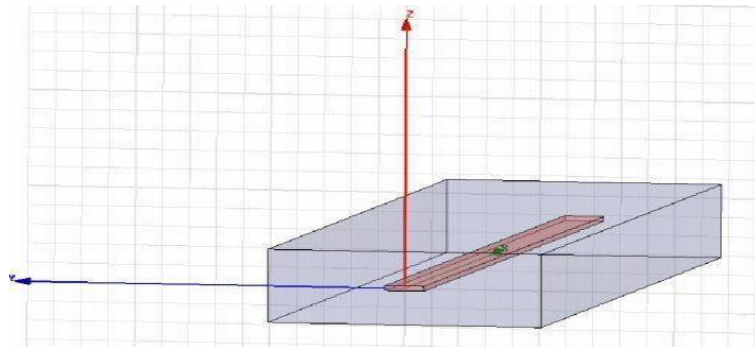
Therefore $Z_{TE} > \eta$

For TEM waves between parallel planes the cut-off frequency is zero and wave impedance for TEM wave is the free space impedance itself

$$Z_{TEM} = \eta$$

Microstrip Line

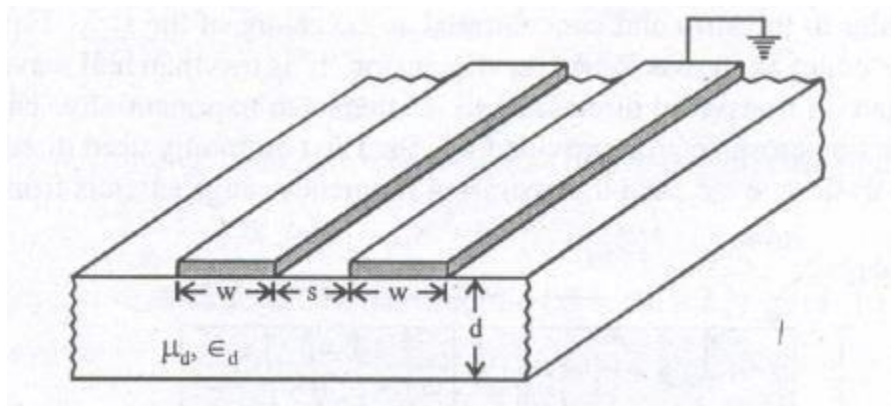
Microstrip Line is an unsymmetrical stripline that is nothing but a parallel plate transmission line having dielectric substrate, the one face of which is metallised ground and the other face has a thin conducting strip of certain width 'w' and thickness 't' some times a cover plate is used for shielding purposes but it is kept much farther away than the ground plane so as not to affect the microstrip field lines.



Stripline equations

A simplified equation for characteristic impedance of stripline is given as:

$$Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln \left[\frac{4H}{0.67\pi W \left(0.8 + \frac{t}{D} \right)} \right]$$



The characteristic impedance of the coplanar strip line is given by

LOSSES IN STRIP LINES:

For low-loss dielectric substrate, the attenuation factor in the strip line arises from conductor losses and is given by

$$\alpha_c = \frac{R_s}{Z_0 b} \frac{(\pi w/b) + \ln(4b/\pi t)}{\ln 2 + (\pi W/2b)} \text{ nepers/unit length}$$

where $R_s = \sqrt{\pi f \mu / \sigma}$

The attenuation constant of a microstrip line depends on frequency of operation, electrical properties of substrate and the conductors and the geometry of mounting of strip on the dielectric.

When the dielectric substrate of dielectric constant is purely non-magnetic then three types of losses occur in microstrip lines. They are

1. Dielectric losses in substrate

1. Ohmic losses in strip conductor and ground plane

1. Radiation loss

Dielectric losses in substrate:

All dielectric materials possess some conductivity but it will be small, but when it is not negligible, then the displacement current density leads the conduction current density by 90 degrees, introducing loss tangent for a lossy dielectric.

1. Ohmic losses in strip conductor and ground plane

In a microstrip line the major contribution to losses at micro frequencies is from finite conductivity of microstrip conductor placed on a low loss dielectric substrate. Due to current flowing through the strip, there will be ohmic losses and hence attenuation of the microwave signal takes place. The current distribution in the transverse plane is fairly uniform with minimum value at the central axis and shooting up to maximum values at the edge of the strip.

2. Radiation losses:

At microwave frequencies, the microstrip line acts as an antenna radiating a small amount of power resulting in radiation losses. This loss depends on the thickness of the substrate, the characteristic impedance Z , effective dielectric constant and the frequency of operation.

For low-loss dielectric substrate, the attenuation factor in the strip line arises from conductor losses and is given by

$$\alpha_c = \frac{R_s}{Z_0 b} \frac{(\pi w/b) + \ln(4b/\pi t)}{\ln 2 + (\pi W/2b)} \text{ nepers/unit length}$$

where $R_s = \sqrt{\pi f \mu / \sigma}$

Cavity Resonators

A cavity resonator is a metallic enclosure that confines the electromagnetic energy i.e. when one end of the waveguide is terminated in a shorting plate there will be reflections and hence standing waves. When another shorting plate is kept at a distance of a multiple of $\lambda_g/2$ than the hollow space so formed can support a signal which bounces back and forth between the two shorting plates. This results in resonance and hence the hollow space is called “cavity” and the resonator as the ‘cavity resonator’

The waveguide section can be rectangular or circular.

The microwave cavity resonator is similar to a tuned circuit at low frequencies having a resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

The cavity resonator can resonate at only one particular frequency like a parallel resonant circuit.

From the figure

$$d = 3\lambda_g/2$$

The stored electric and magnetic energies inside the cavity determine its equivalent inductance and capacitance.

The energy dissipated by the finite conductivity of the cavity walls determines its equivalent resistance.

A given resonator has an infinite number of resonant modes and each mode corresponds to a definite resonant frequency.

When the frequency of an impressed signal is equal to a resonant frequency a maximum amplitude of the standing wave occurs and the peak energies stored in the electric and magnetic fields are equal.

The mode having the lowest resonant frequency is called as the 'Dominant mode'

Rectangular cavity Resonator

The electromagnetic field inside the cavity should satisfy Maxwell's equations subject to the boundary conditions that the electric field tangential to and the magnetic field normal to the metal walls must vanish.

The wave equations in the rectangular resonator should satisfy the boundary condition of the zero tangential 'E' At four of the walls.

(1) TE waves

For a TE wave $E_z=0, H_z \neq 0$

From Maxwell's equation

$$\nabla^2 H_z = -\omega^2 \mu \epsilon H_z$$

$$\partial^2 H_z / \partial x^2 + \partial^2 H_z / \partial y^2 + \partial^2 H_z / \partial z^2 = -\omega^2 \mu \epsilon H_z$$

Since $\partial^2 / \partial z^2 = \gamma^2$

$$\partial^2 H_z / \partial x^2 + \partial^2 H_z / \partial y^2 + (\gamma^2 + \omega^2 \mu \epsilon) H_z = 0$$

Let $\gamma^2 + \omega^2 \mu \epsilon = h^2$

$$\partial^2 H_z / \partial x^2 + \partial^2 H_z / \partial y^2 + h^2 H_z = 0 \text{-----(1)}$$

This is a partial differential equation of 2nd order

$$\text{Let } H_z = XY \text{-----(2)}$$

Where X is a function of 'x' alone, Y is a function of 'y' alone

$$Y \partial^2 X / \partial x^2 + X \partial^2 Y / \partial y^2 + h^2 XY = 0$$

$$1/X \partial^2 X / \partial x^2 + 1/Y \partial^2 Y / \partial y^2 + h^2 = 0 \text{-----(3)}$$

Where h^2 is a constant since γ^2 and $\omega^2 \mu \epsilon$ are constants.

So to satisfies the above equation sum of functions of 'X' and 'Y' must be equalent to a constant. It is possible when individual one must be a constant.

$$\text{Let } 1/X \partial^2 X / \partial x^2 = -B^2, 1/Y \partial^2 Y / \partial y^2 = -A^2 \text{-----(4)}$$

Where A^2 and B^2 are constants

$$-A^2 - B^2 + h^2 = 0$$

$$h^2 = A^2 + B^2 \text{-----(5)}$$

solutions of equation (4) are

$$X = c_1 \cos Bx + c_2 \sin Bx \text{-----(6)}$$

$$Y = c_3 \cos Ay + c_4 \sin Ay \text{-----(7)}$$

Where c_1, c_2, c_3 and c_4 are constants

Which are determined by applying boundary conditions

i) Boundary condition(Bottom wall)

$E_x = 0$ for $y=0$ and all values of x varying from 0 to a we know

$$E_x = -\gamma / h^2 \partial E_z / \partial x - j\omega \mu / h^2 \partial H_z / \partial y$$

Since $E_z = 0$

$$E_x = -j\omega \mu / h^2 \partial H_z / \partial y$$

$$E_x = -j\omega \mu / h^2 \partial / \partial y [(c_1 \cos Bx + c_2 \sin Bx)(c_3 \cos Ay + c_4 \sin Ay)]$$

$$= -j\omega \mu / h^2 [(c_2 \sin Bx + c_1 \cos Bx)(-Ac_3 \sin Ay + Ac_4 \cos Ay)]$$

From the boundary condition (i)

$$0 = -j\omega \mu / h^2 [c_1 \cos Bx + c_2 \sin Bx] Ac_4$$

$$[c_1 \cos Bx + c_2 \sin Bx] A c_4 = 0$$

$$A \neq 0, c_1 \cos Bx + c_2 \sin Bx \neq 0$$

$$c_4 = 0$$

$$H_z = (c_1 \cos Bx + c_2 \sin Bx)(c_3 \cos Ay)$$

$$H_z = c_3(c_1 \cos Bx + c_2 \sin Bx) \cos Ay$$

ii) 2nd Boundary condition [left side wall]

$$E_y = 0 \text{ for } x=0 \text{ and } y \text{ varying from } 0 \text{ to } b.$$

$$\text{Since } E_y = j\omega\mu/h^2 \partial H_z / \partial x$$

$$E_y = j\omega\mu/h^2 \partial / \partial x [(c_1 \cos Bx + c_2 \sin Bx) c_3 \cos Ay]$$

$$E_y = j\omega\mu/h^2 [-Bc_1 \sin Bx + Bc_2 \cos Bx] c_3 \cos Ay$$

From the boundary condition

$$0 = j\omega\mu/h^2 [Bc_2 c_3 \cos Bx \cos Ay]$$

$$c_3 \cos Ay \neq 0, c_2 = 0$$

$$H_z = c_1 \cos Bx \cdot c_3 \cos Ay$$

iii) 3rd Boundary condition [Top wall]

$$E_x = 0 \text{ for } y=b \text{ and } x \text{ varies from } 0 \text{ to } a$$

$$E_x = -j\omega\mu/h^2 \partial H_z / \partial y$$

$$E_x = -j\omega\mu/h^2 \partial [c_1 c_3 \cos Bx \cos Ay] / \partial y = j\omega\mu/h^2 c_1 c_3 A \cos Bx \sin Ay$$

$$c_1 c_3 \cos Bx \sin Ay b = 0$$

$$Ab = n\pi \text{ where } n=0,1,2,3, \dots$$

$$A = n\pi/b$$

iv) 4th Boundary condition [Right side wall]

$$E_y = 0 \text{ for } x=a \text{ and } y \text{ varying from } 0 \text{ to } a$$

$$E_y = j\omega\mu/h^2 \partial / \partial x [c_1 c_3 \cos Bx \cos Ay] = j\omega\mu/h^2 c_1 c_3 B \sin Bx \cos Ay$$

$$E_y = j\omega\mu/h^2 c_1 c_3 B \sin Bx \cos Ay$$

From the boundary condition $E_y = 0$ for $x=a$ and y varying from 0 to a

$$c_1 c_3 \sin Ba \cos Ay = 0$$

$$\sin Ba = 0$$

$$Ba = m\pi$$

$$B = m\pi/a$$

$$c_1 c_3 = c$$

$$H_z = c \cos(m\pi/a)x \cos(n\pi/b)y e^{j(\omega t - \beta z)}$$

The amplitude constant along the positive 'z' direction is represented by A^+ , and that along the negative 'z' direction by A^- .

Adding the two travelling waves to obtain the fields of standing wave when we have from above equation.

$$H_z = (A^+ e^{-j\beta z} + A^- e^{j\beta z}) \cos(m\pi/a)x \cos(n\pi/b)y e^{j\omega t}$$

To make E_y vanish at $z=0$ and $z=d$ we must $A^+ = A^-$

$$E_y = -\gamma/h^2 \partial E_z / \partial y + j\omega\mu/h^2 \partial H_z / \partial x \quad \text{since } E_z = 0$$

$$E_y = j\omega\mu/h^2 \partial (A^+ e^{-j\beta z} + A^- e^{j\beta z}) \cos(m\pi/a)x \cos(n\pi/b)y e^{j\omega t} / \partial x$$

$$0 = [(A^+ e^{-j\beta z} + A^- e^{j\beta z}) - \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y] e^{j\omega t}$$

$$\text{But } \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \neq 0$$

$$\text{Therefore } A^+ e^{-j\beta z} + A^- e^{j\beta z} = 0$$

To make $E_y = 0$, $A^+ = -A^-$

$$A^+ [e^{-j\beta z} - e^{j\beta z}] = 0$$

$$-2j \sin \beta z \cdot A^+ = 0$$

$A^+ \neq 0$ only $\sin \beta d = 0$ with $z = d$

$$\beta d = p\pi$$

$$\beta = p\pi/d$$

$$H_z = c \cos(m\pi/a)x \cos(n\pi/b)y \sin(p\pi/d)z e^{j(\omega t - \beta z)}$$

Resonant Frequency(f_0)

Since we know that for a rectangular waveguide

$$h^2 = \gamma^2 + \omega^2 \mu \epsilon = A^2 + B^2 = (m\pi/a)^2 + (n\pi/b)^2$$

$$\omega^2 \mu \epsilon = (m\pi/a)^2 + (n\pi/b)^2 - \gamma^2$$

for a wave propagation $\gamma = j\beta$

$$\text{or } \gamma^2 = -\beta^2$$

$$\omega^2 \mu \epsilon = (m\pi/a)^2 + (n\pi/b)^2 + \beta^2$$

if a wave has to exist in a cavity resonator there must be a phase change corresponding to a given guide wavelength. The condition for the resonator to resonate is $\beta = p\pi/d$.

where p = a constant 1, 2, 3, ..., ∞ , that indicates half wave variation of either electric or magnetic field along that z-direction

d=length of the resonator

when $\beta = p\pi/d$, $f = f_c$ $\omega = \omega_0$

$$\omega_0^2 \mu \epsilon = (m\pi/a)^2 + (n\pi/b)^2 + (p\pi/d)^2$$

$$(2\pi f_0)^2 \mu \epsilon = (m\pi/a)^2 + (n\pi/b)^2 + (p\pi/d)^2$$

$$f_0 = \frac{1}{2\pi\sqrt{\mu\epsilon}} [(m\pi/a)^2 + (n\pi/b)^2 + (p\pi/d)^2]^{1/2}$$

$$f_0 = \frac{c}{2} [(m/a)^2 + (n/b)^2 + (p/d)^2]^{1/2}$$

General mode of propagation in a cavity resonator is TE_{mnp} or TM_{mnp} .

For both TE and TM the resonant frequency is the same in a rectangular cavity resonator.

Dominant Mode

The mode for which cut-off wavelength is large is called Dominant Mode.

For $a > b < d$ the dominant mode is the TE_{101} mode.

Q-Factor

The quality factor 'Q' is a measure of the frequency selectivity of a resonator or antiresonant circuit and it is defined as

$$Q = 2\pi \frac{\text{Maximum energy stored per cycle}}{\text{Energy dissipated per cycle}} = \omega W/P$$

The Q of a perfect or ideal cavity resonator is infinite since in a perfect conductor forming the cavity P would be zero and also once energized it would resonate for ever.

If there is more than one resonate frequency there will be different values of Q for the various values of frequencies.

Coupling loops are used to couple the energy in and out of a cavity resonator. This coupling changes the value of Q. that to into account the coupling between the cavity and coupling path is known as the loaded Q_L . So, Q_L can be given by

$$1/Q_L = 1/Q_0 + 1/Q_{\text{ext}}$$

There are three types of couplings they are critically coupling, under coupling, over coupling.

An unloaded resonator can be represented by a series or parallel resonant circuit. The unloaded Q is then given by

$$Q_0 = \omega_0 L/R$$

Then Q_{ext} can be written as

$$Q_{\text{ext}} = Q_0/k = \omega_0 L / K r$$

$$Q_L = Q_0 / (1 + k)$$

For critical coupled cavity is given by

$$Q_L = 1/2 Q_{\text{ext}} = 1/2 Q_0$$

For under coupled cavity $k < 1$ the cavity terminals are at a voltage minimum and the input impedance is the reciprocal of standing wave ratio.

$$\text{i.e. } k = 1/\rho$$

$$\text{i.e. } Q_L = \frac{\rho}{1 + \rho}$$

For over coupled cavity $k > 1$ cavity terminals are at a voltage maximum and the impedance is standing wave ratio.

$$\text{i.e. } k = \rho$$

$$\text{i.e. } Q_L = Q_0 / (1 + \rho)$$

UNIT- II

MICROWAVE WAVEGUIDE COMPONENTS AND APPLICATIONS

INTRODUCITON

WAVE GUIDE CORNERS , BENDS AND TWISTS:

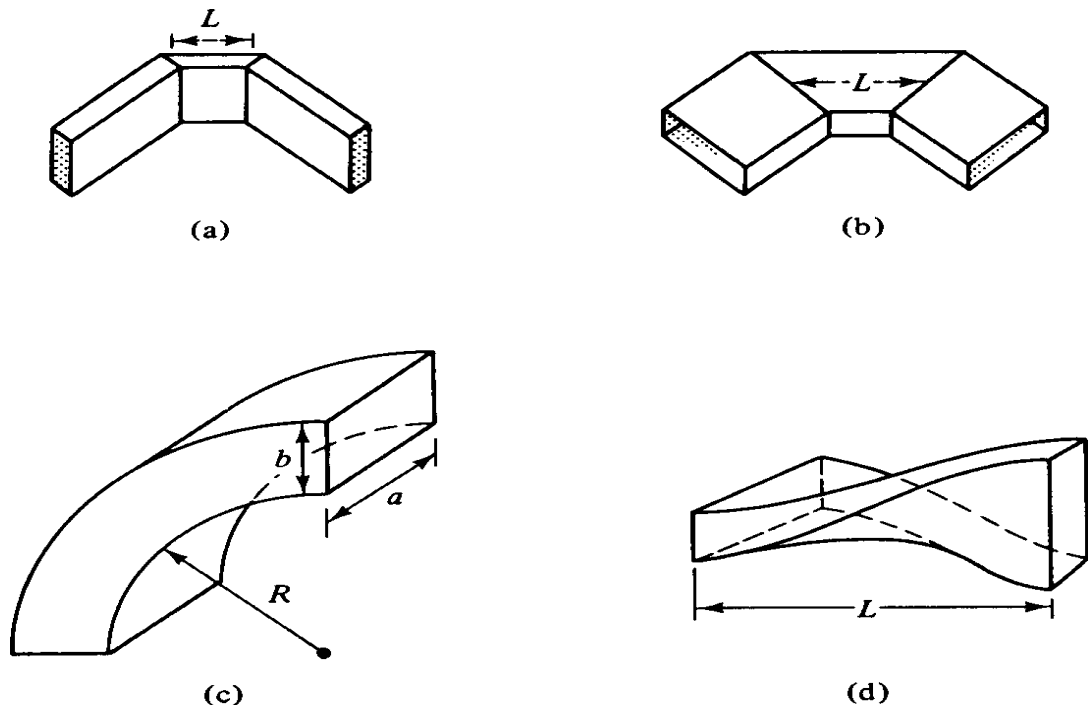
The waveguide corner, bend, and twist are shown in figure below, these waveguide components are normally used to change the direction of the guide through an arbitrary angle.

In order to minimize reflections from the discontinuities, it is desirable to have the mean length L between continuities equal to an odd number of quarter wave lengths. That is,

$$L = (2n + 1) \frac{\lambda_g}{4}$$

where $n = 0, 1, 2, 3, \dots$, and λ_g is the wavelength in the waveguide. If the mean length L is an odd number of quarter wavelengths, the reflected waves from both ends of the waveguide section are completely canceled. For the waveguide bend, the minimum radius of curvature for a small reflection is given by Southworth as

$$R = 1.5b \quad \text{for an } E \text{ bend} \quad R = 1.5a \quad \text{for an } H \text{ bend}$$



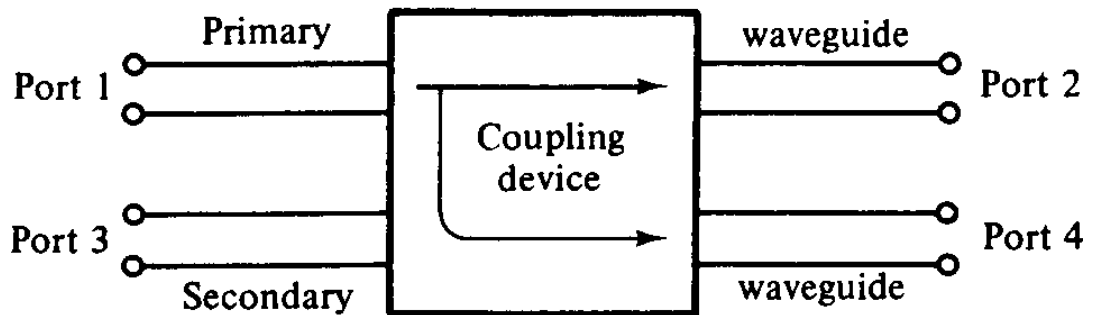
Waveguide corner, bend, and twist. (a) *E*-plane corner. (b) *H*-plane corner. (c) Bend. (d) Continuous twist.

DIRECTIONAL COUPLERS:

A directional coupler is a four-port waveguide junction as shown below. It Consists of a primary waveguide 1- 2 and a secondary waveguide 3-4. When all Ports are terminated in their characteristic impedances, there is free transmission of the waves without reflection, between port 1 and port 2, and there is no transmission of power between port 1 and port 3 or between port 2 and port 4 because no coupling exists between these two pairs of ports. The degree of coupling between port 1 and port4 and between port 2 and port 3 depends on the structure of the coupler.

The characteristics of a directional coupler can be expressed in terms of its Coupling factor and its directivity.

Assuming that the wave is propagating from portto port2 in the primary line, the coupling factor and the directivity are defined,



Directional coupler.

where P_1 = power input to port 1

P_3 = power output from port 3

P_4 = power output from port 4

$$\text{Coupling factor (dB)} = 10 \log_{10} \frac{P_1}{P_4}$$

$$\text{Directivity (dB)} = 10 \log_{10} \frac{P_4}{P_3}$$

It should be noted that port 2, port 3, and port 4 are terminated in their characteristic impedances. The coupling factor is a measure of the ratio of power levels in the primary and secondary lines. Hence if the coupling factor is known, a fraction of power measured at port 4 may be used to determine the power input at port 1 .

This significance is desirable for microwave power measurements because no disturbance, which may be caused by the power measurements, occurs in the primary line. The directivity is a measure of how well the forward traveling wave in the primary waveguide couples only to a specific port of the secondary waveguide ideal directional coupler should have infinite directivity. In other words, the power at port 3 must be zero because port 2 and portA are perfectly matched. Actually well-designed directional couplers have a directivity of only 30 to 35 dB.

Several types of directional couplers exist, such as a two-hole direct couler, four-hole directional coupler, reverse-coupling directional coupler , and Bethe- hole directional coupler the very commonly used two-hole directional coupler is described here.

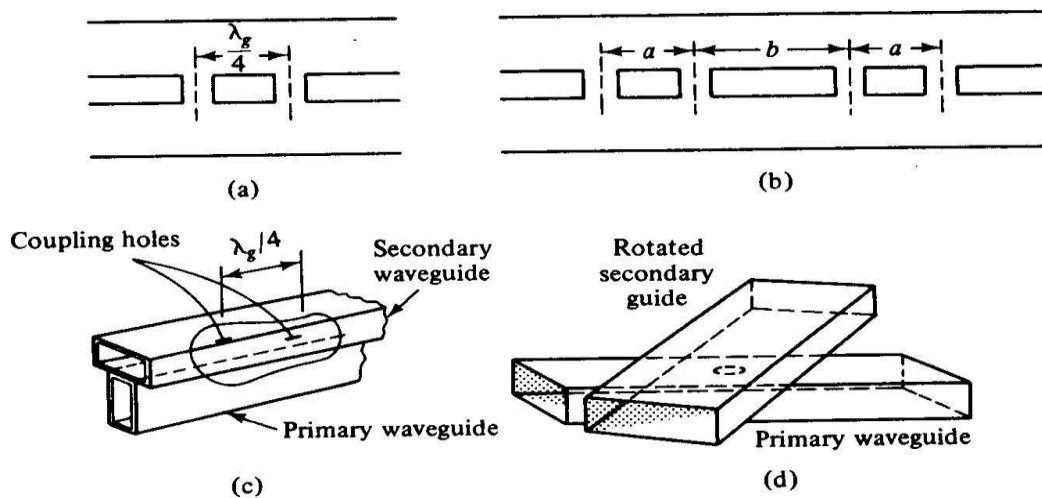


Figure 4-5-2 Different directional couplers. (a) Two-hole directional coupler. (b) Four-hole directional coupler. (c) Schwinger coupler. (d) Bethe-hole directional coupler.

TWO HOLE DIRECTIONAL COUPLERS:

A two hole directional coupler with traveling wave propagating in it is illustrated . the spacing between the centers of two holes is

$$L = (2n + 1) \frac{\lambda_g}{4}$$

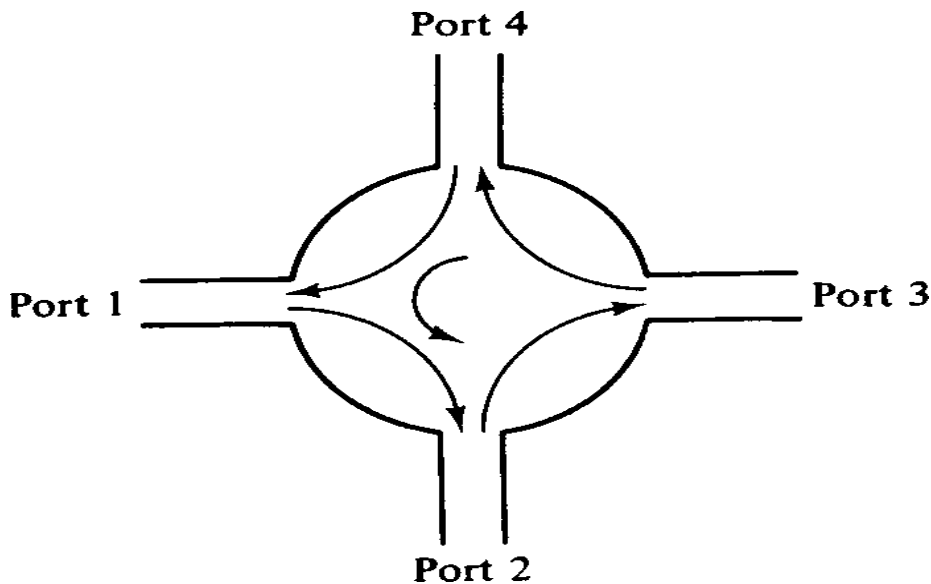
A fraction of the wave energy entered into port 1 passes through the holes and is radiated into the secondary guide as the holes act as slot antennas. The forward waves in the secondary guide are in same phase, regardless of the hole space and are added at port 4. the backward waves in the secondary guide are out of phase and are cancelled in port 3.

CIRCUALTORS AND ISOLATORS:

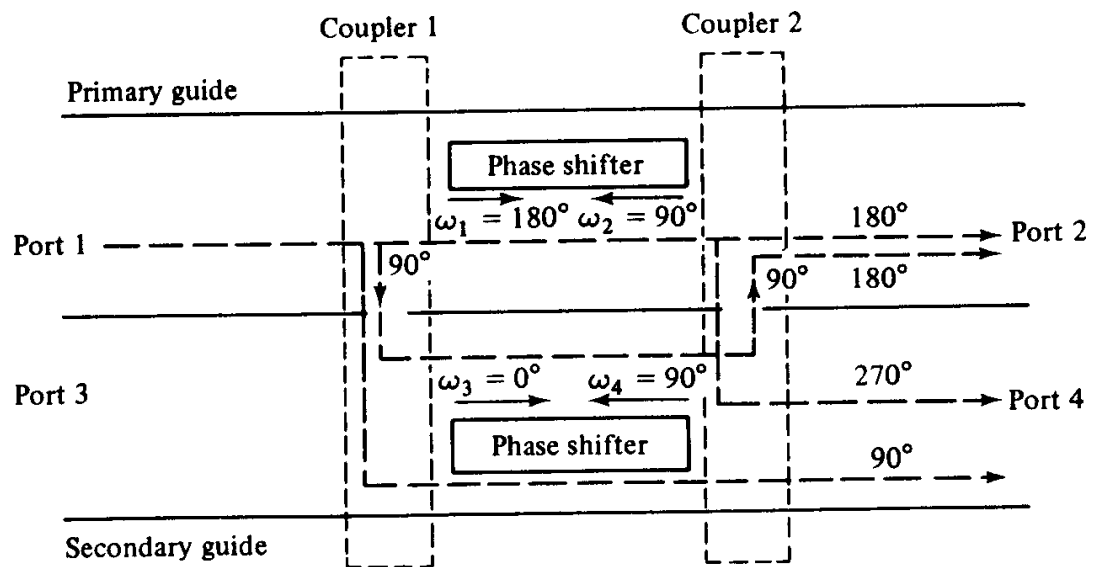
Both microwave circulators and isolators are non reciprocal transmission devices that use the property of Faraday rotation in the ferrite material. A non reciprocal phase shifter consists of thin slab of ferrite placed in a rectangular waveguide at a point where the dc magnetic field of the incident wave mode is circularly polarized. When a piece of ferrite is affected by a dc magnetic field the ferrite exhibits Faraday rotation. It does so because the ferrite is nonlinear material and its permeability is an asymmetric tensor.

MICROWAVE CIRCULATORS:

A *microwave circulator* is a multiport waveguide junction in which the wave can flow only from the n th port to the $(n + 1)$ th port in one direction Although there is no restriction on the number of ports, the four-port microwave circulator is the most common. One type of four-port microwave circulator is a combination of two 3-dB side hole directional couplers and a rectangular waveguide with two non reciprocal phase shifters.



The symbol of a circulator.



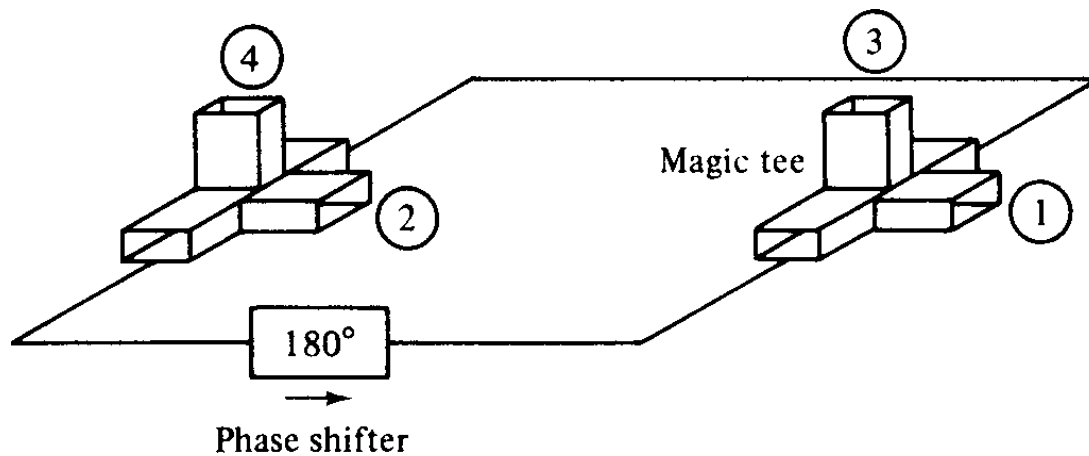
Schematic diagram of four-port circulator.

The operating principle of a typical microwave circulator can be analyzed with the aid of Fig shown above. Each of the two 3- dB couplers in the circulator introduces a phase shift of 90° , and each of the two phase shifters produces a certain amount of phase change in a certain direction as indicated. When a wave is incident to port 1, the wave is split into two components by coupler I. The wave in the primary guide arrives at port 2 with a relative phase change of 180° . The second wave propagates through the two couplers and the secondary guide and arrives at port 2 with a relative phase shift of 180° . Since the two waves reaching port 2 are in phase, the power transmission is obtained from port 1 to port 2. However, the wave propagates through the primary guide, phase shifter, and coupler 2 and arrives at port 4 with a phase change of 270° . The wave travels through coupler 1 and the secondary guide, and it arrives at port 4 with a phase shift of 90° . Since the two waves reaching port 4 are out of phase by 180° , the power transmission from port 1 to port 4 is zero. In general, the differential propagation constants in the two directions of propagation in a waveguide containing ferrite phase shifters should be

$$\omega_1 - \omega_3 = (2m + 1)\pi \quad \text{rad/s}$$

$$\omega_2 - \omega_4 = 2n\pi \quad \text{rad/s}$$

where m and n are any integers, including zeros. A similar analysis shows that a wave incident to port 2 emerges at port 3 and so on. As a result, the sequence of power flow is designated as $1 \sim 2 \sim 3 \sim 4 \sim 1$. Many types of microwave circulators are in use today. However, their principles of operation remain the same. A four-port circulator is constructed by the use of two magic tees and a phase shifter. The phase shifter produces a phase shift of 180° .



A four-port circulator.

A perfectly matched, lossless, and nonreciprocal four-port circulator has an S matrix of the form

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{21} & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & S_{34} \\ S_{41} & S_{42} & S_{43} & 0 \end{bmatrix}$$

Using the properties of S parameters the S-matrix is

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

MICROWAVE ISOLATORS:

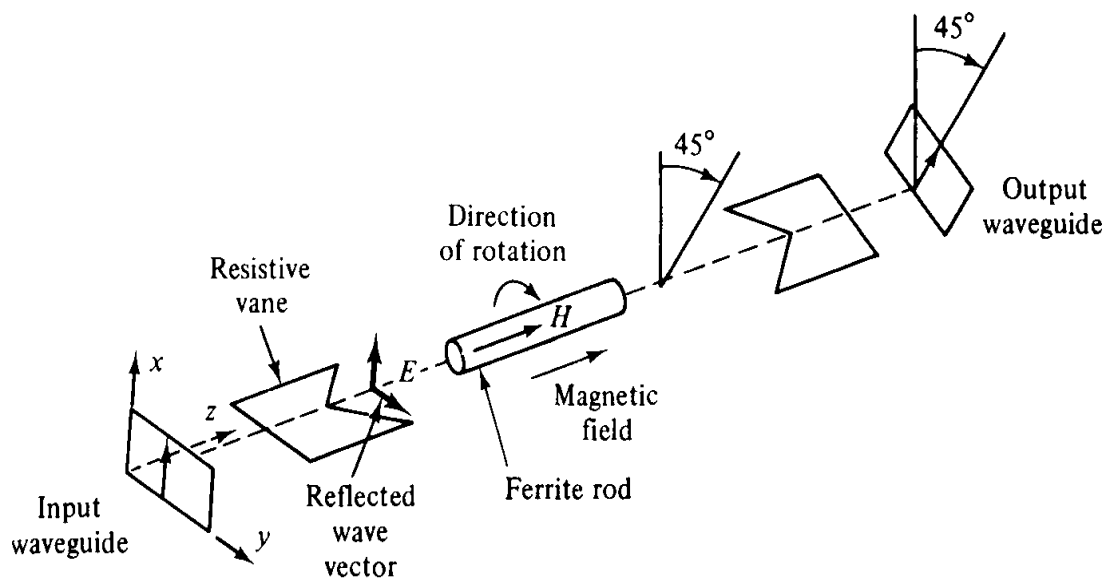
An *isolator* is a nonreciprocal transmission device that is used to isolate one component from reflections of other components in the transmission line. An ideal isolator completely absorbs the power for propagation in one direction and provides lossless transmission in the opposite direction.

Thus the isolator is usually called *uniline*.

Isolators are generally used to improve the frequency stability of microwave generators, such as klystrons and magnetrons, in which the reflection from the load affects the generating frequency. In such cases, the isolator placed between the generator and load prevents the reflected power from the unmatched load from returning to the generator. As a result, the isolator maintains the frequency stability of the generator.

Isolators can be constructed in many ways. They can be made by terminating ports 3 and 4 of a four-port circulator with matched loads. On the other hand, isolators can be made by inserting a ferrite rod along the axis of a rectangular waveguide as shown below.

The isolator here is a Faraday-rotation isolator. Its operating principle can be explained as follows . The input resistive card is in the y - z plane, and the output resistive card is displaced 45° with respect to the input card. The dc magnetic field, which is applied longitudinally to the ferrite rod, rotates the wave plane of polarization by 45° . The degrees of rotation depend on the length and diameter of the rod and on the applied dc magnetic field. An input TE₁₀ dominant mode is incident to the left end of the isolator. Since the TE₁₀ mode wave is perpendicular to the input resistive card, the wave passes through the ferrite rod without attenuation. The wave in the ferrite rod section is rotated clockwise by 45° and is normal to the output resistive card. As a result of rotation, the wave arrives at the output.



end without attenuation at all. On the contrary, a reflected wave from the output end is similarly rotated clockwise 45° by the ferrite rod. However, since the reflected wave is parallel to the input resistive card, the wave is thereby absorbed by the input card. The typical performance of these isolators is about 1-dB insertion loss in forward transmission and about 20- to 30-dB isolation in reverse attenuation.

WAVE GUIDE TEE JUNCTIONS:

A waveguide Tee is formed when three waveguides are interconnected in the form of English alphabet T and thus waveguide tee is 3-port junction. The waveguide tees are used to connect a branch or section of waveguide in series or parallel with the main waveguide transmission line either for splitting or combining power in a waveguide system.

There are basically 2 types of tees namely

1.H- plane Tee junction

2.E-plane Tee junction

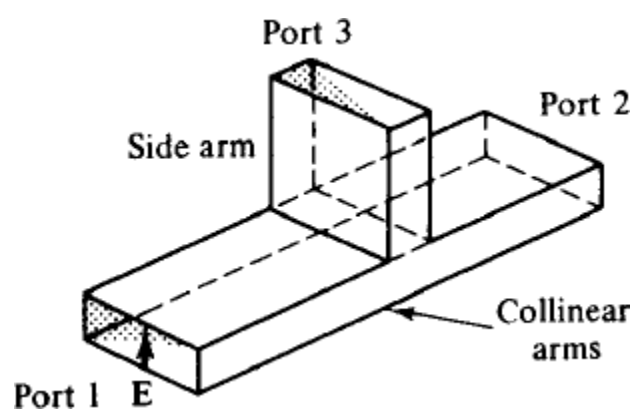
A combination of these two tee junctions is called a hybrid tee or “ Magic Tee”.

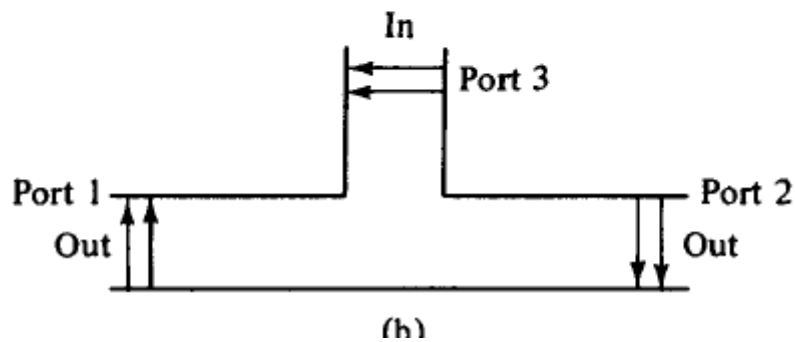
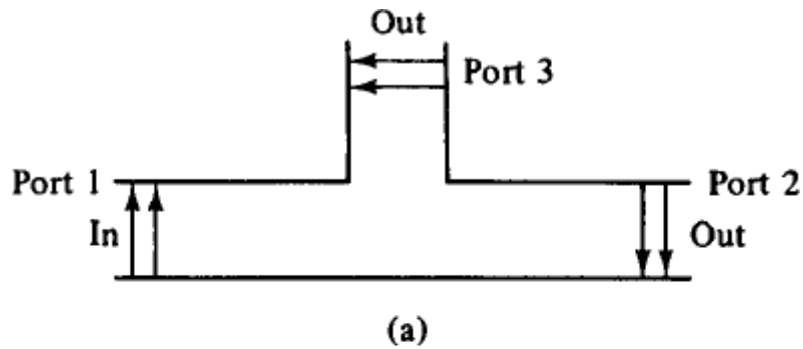
E-plane Tee(series tee):

An E-plane tee is a waveguide tee in which the axis of its side arm is parallel to the E field of the main guide . if the collinear arms are symmetric about the side arm.

If the E-plane tee is perfectly matched with the aid of screw tuners at the junction , the diagonal components of the scattering matrix are zero because there will be no reflection.

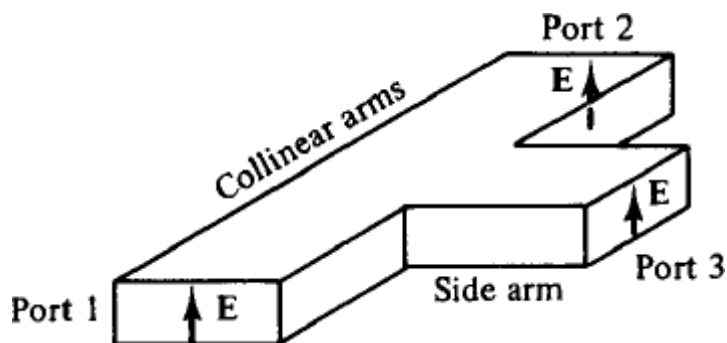
When the waves are fed into side arm, the waves appearing at port 1 and port 2 of the collinear arm will be in opposite phase and in same magnitude.





H-plane tee: (shunt tee)

An H-plane tee is a waveguide tee in which the axis of its side arm is shunting the E field or parallel to the H-field of the main guide.

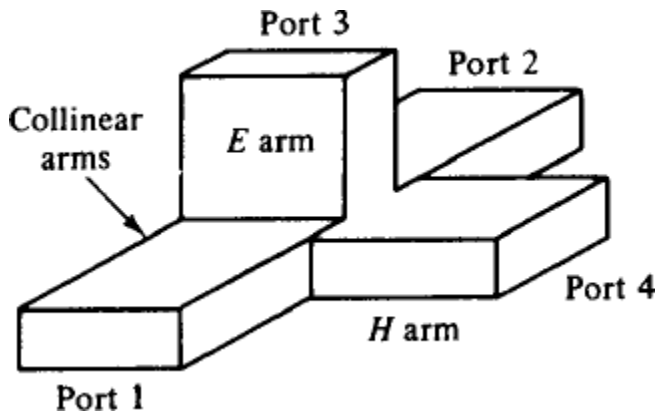


If two input waves are fed into port 1 and port 2 of the collinear arm, the output wave at port 3 will be in phase and additive .

If the input is fed into port 3, the wave will split equally into port 1 and port 2 in phase and in same magnitude .

Magic Tee (Hybrid Tees)

A magic tee is a combination of E-plane and H-plane tee. The characteristics of magic tee are:



1. If two waves of equal magnitude and same phase are fed into port 1 and port 2 the output will be zero at port 3 and additive at port 4.
2. If a wave is fed into port 4 it will be divided equally between port 1 and port 2 of the collinear arms and will not appear at port 3.
3. If a wave is fed into port 3, it will produce an output of equal magnitude and opposite phase at port 1 and port 2. the output at port 4 is zero.
4. If a wave is fed into one of the collinear arms at port 1 and port 2, it will not appear in the other collinear arm at port 2 or 1 because the E-arm causes a phase delay while H arm causes a phase advance.

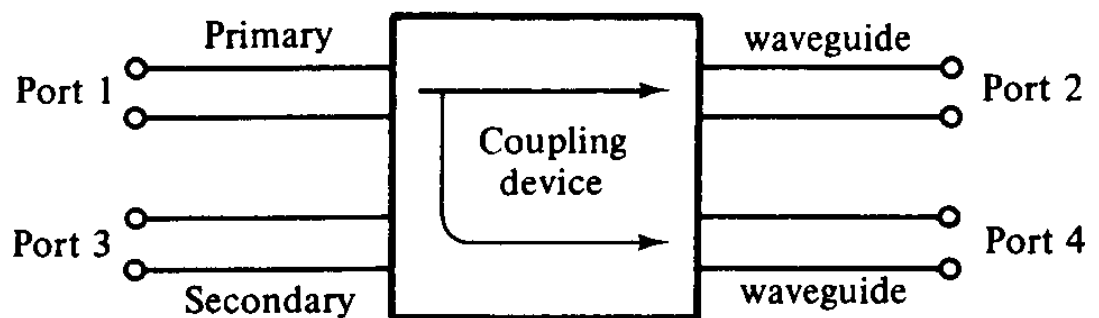
Therefore the **S** matrix of a magic tee can be expressed as

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & 0 \\ S_{41} & S_{42} & 0 & 0 \end{bmatrix}$$

DIRECTIONAL COUPLERS:

A directional coupler is a four-port waveguide junction as shown below. It Consists of a primary waveguide 1-2 and a secondary waveguide 3-4. When all Ports are terminated in their characteristic impedances, there is free transmission of the waves without reflection, between port 1 and port 2, and there is no transmission of power between port 1 and port 3 or between port 2 and port 4 because no coupling exists between these two pairs of ports. The degree of coupling between port 1 and port4 and between port 2 and port 3 depends on the structure of the coupler.

The characteristics of a directional coupler can be expressed in terms of its Coupling factor and its directivity. Assuming that the wave is propagating from port 1 to port 2 in the primary line, the coupling factor and the directivity are defined,



Directional coupler.

where P_1 = power input to

port 1 P_3 = power output

$$\text{Coupling factor (dB)} = 10 \log_{10} \frac{P_1}{P_4}$$

$$\text{Directivity (dB)} = 10 \log_{10} \frac{P_4}{P_3}$$

from port 3 $P_4 =$ power

output from port 4

It should be noted that port 2, port 3, and port 4 are terminated in their characteristic impedances. The coupling factor is a measure of the ratio of power levels in the primary and secondary lines. Hence if the coupling factor is known, a fraction of power measured at port 4 may be used to determine the power input at port 1.

This significance is desirable for microwave power measurements because no disturbance, which may be caused by the power measurements, occurs in the primary line. The directivity is a measure of how well the forward traveling wave in the primary waveguide couples only to a specific port of the secondary waveguide ideal directional coupler should have infinite directivity. In other words, the power at port 3 must be zero because port 2 and port 4 are perfectly matched. Actually well- designed directional couplers have a directivity of only 30 to 35 dB. Several types of directional couplers exist, such as a two-hole directional coupler, four-hole directional coupler, reverse- coupling directional coupler , and Bethe-hole directional coupler the very commonly used two-hole directional coupler is described here.

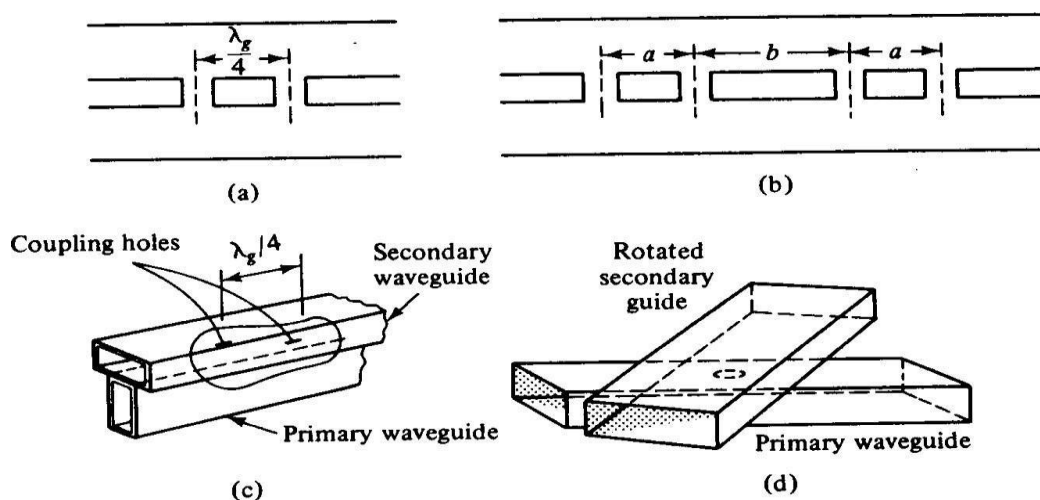


Figure 4-5-2 Different directional couplers. (a) Two-hole directional coupler. (b) Four-hole directional coupler. (c) Schwinger coupler. (d) Bethe-hole directional coupler.

TWO HOLE DIRECTIONAL COUPLERS:

A two hole directional coupler with traveling wave propagating in it is illustrated . the spacing between the centers of two holes is

$$L = (2n + 1) \frac{\lambda_g}{4}$$

A fraction of the wave energy entered into port 1 passes through the holes and is radiated into the secondary guide as the holes act as slot antennas. The forward waves in the secondary guide are in same phase, regardless of the hole space and are added at port 4. the backward waves in the secondary guide are out of phase and are cancelled in port 3.

S-matrix for Directional coupler:

The following characteristics are observed in an ideal Directional Coupler:

1. Since the directional coupler is a 4-port junction, the order or (S I matrix is 4 x 4 given by

$$[S]_{DC} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

2. Microwave power fed into port (1) cannot come out of port (3) as port (3) is the back port. Therefore the scattering co-efficient S_{13} is zero...

$$S_{13} = 0$$

3. Because of the symmetry of the junction, an input power at port (2) cannot couple to port (4) as port (4) is the back-port for port (2)

$$S_{24} = 0$$

4. Let us assume that port (3) and (4) are perfectly matched to the junction so that

$$S_{33} = S_{44} = 0$$

Then, the remaining two ports will be "automatically" matched to the junction

$$S_{11} = S_{22} = 0$$

From the symmetric property of ISI matrix, we have

$$S_{ij} = S_{ji}$$

With the above characteristic values for S-parameters, the matrix of (5.125)

$$[S]_x = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}$$

becomes

From unitary property of equation we have

$$[\text{Since } S_{21} = S_{12}, S_{31} = S_{13} = 0, S_{32} = S_{23}, S_{41} = S_{14}, S_{42} = S_{24} = 0 \text{ and } S_{43} = S_{34}]$$

$$[S] [S]^* = [U]$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Considering 1st row and 1st column,

$$|S_{12}|^2 + |S_{14}|^2 = 1$$

Considering 2nd row and 2nd column,

$$|S_{12}|^2 + |S_{23}|^2 = 1$$

Considering 3rd row and 3rd column,

$$|S_{23}|^2 + |S_{34}|^2 = 1$$

Considering 1st row and 3rd column,

$$S_{12} S_{23}^* + S_{14} S_{34}^* = 0$$

Comparison of equations (5.133) and (5.134) yields

$$S_{14} = S_{23}$$

Comparing equations (5.134) and (5.135), we get

$$S_{12} = S_{34}$$

Let S_{12} be "*real and positive*" equal to p

Then $S_{34} = p = S_{34}^* = S_{12}$

Using equations (5.137) and (5.139) in (5.136), we get

$$S_{12} S_{23}^* + S_{23} S_{12} = 0$$

$$\therefore S_{12} (S_{23} + S_{23}^*) = 0$$

Since $S_{12} \neq 0$, we must have $S_{23} + S_{23}^* = 0$

Equation (5.140) will be satisfied only when S_{23} is purely imaginary.

Let $S_{23} = jq = S_{14}$

Using the above obtained values of S-parameters in the matrix of equation (5.131), we get

$$[S]_{DC} = \begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix} \quad (5.142)$$

The relationship between p and q can be obtained from equation (5.133) as

$$p^2 + q^2 = 1 \quad (5.143)$$

The quantity 'p' is called the "**transmission factor**" and 'q' is called the "**coupling factor**".

UNIT-3 MICROWAVE TUBES

Limitations and Losses of conventional tubes at microwave frequencies

Following are the limitations of conventional active devices like transistors or tubes at microwave frequencies

1) Interelectrode capacitance.

What is interelectrode capacitance?

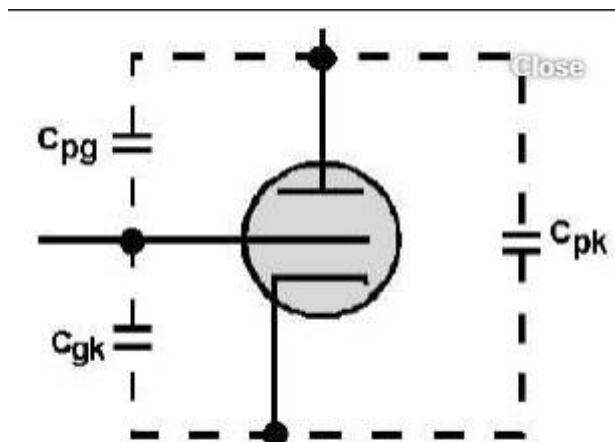
Vacuum has a dielectric constant of 1. As the elements of the triodes are made of metal and are separated by a dielectric, capacitance exists between them. This capacitance is interelectrode capacitance.

The capacitance between the plate and grid is C_{pg} . The grid-to-cathode capacitance is C_{gk} . The total capacitance across the tube is C_{pk} .

Now, we know that the capacitive reactance is given by

So as the input frequency increases, the effective grid to cathode impedance decreases due to decrease in reactance of interelectrode capacitance. At higher frequencies (greater than 100MHz) it becomes so small that signal is short circuited with the tube. Also, gain of the device reduces significantly.

This effect can be minimized by taking smaller (reducing the area) electrodes and by increasing distance between them (i.e. reducing capacitance because $C = \epsilon \cdot A/d$) therefore by increasing reactance.

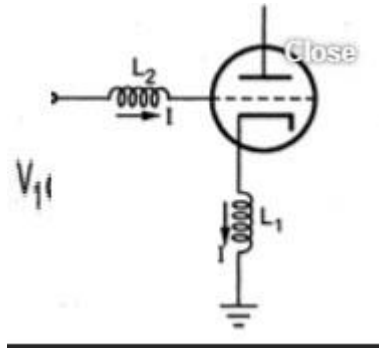


2) Lead inductance.

Lead or stray inductance are effectively in parallel within the device with the interelectrode capacitance. Inductive reactance is given by:

$$X_L = 2 \pi f L$$

As the frequency increases, the effective reactance of the circuit also increases. This effect raises the frequency limit to the device. The inductance of cathode lead is common to both grid and plate circuits. This provides a path for degenerative feedback which reduces the overall efficiency of the circuit.



3) Transit time

Transit time is the time required for electrons to travel from the cathode to the plate. At low frequency, the transit time is very negligible. But, however at higher frequencies, transit time becomes an appreciable portion of a signal cycle which results in decrease in efficiency of device.

4) Gain bandwidth product

Gain bandwidth product is independent of frequency. So for a given tube higher gain can be only obtained at the expense of narrower bandwidth.

5) Skin effect

This effect is introduced at higher frequencies. Due to it, the current flows from the small sectional area to the surface of the device. Also at higher frequencies, resistance of conductor increases due to which there are losses.

$$R = \rho l (\sqrt{f})$$

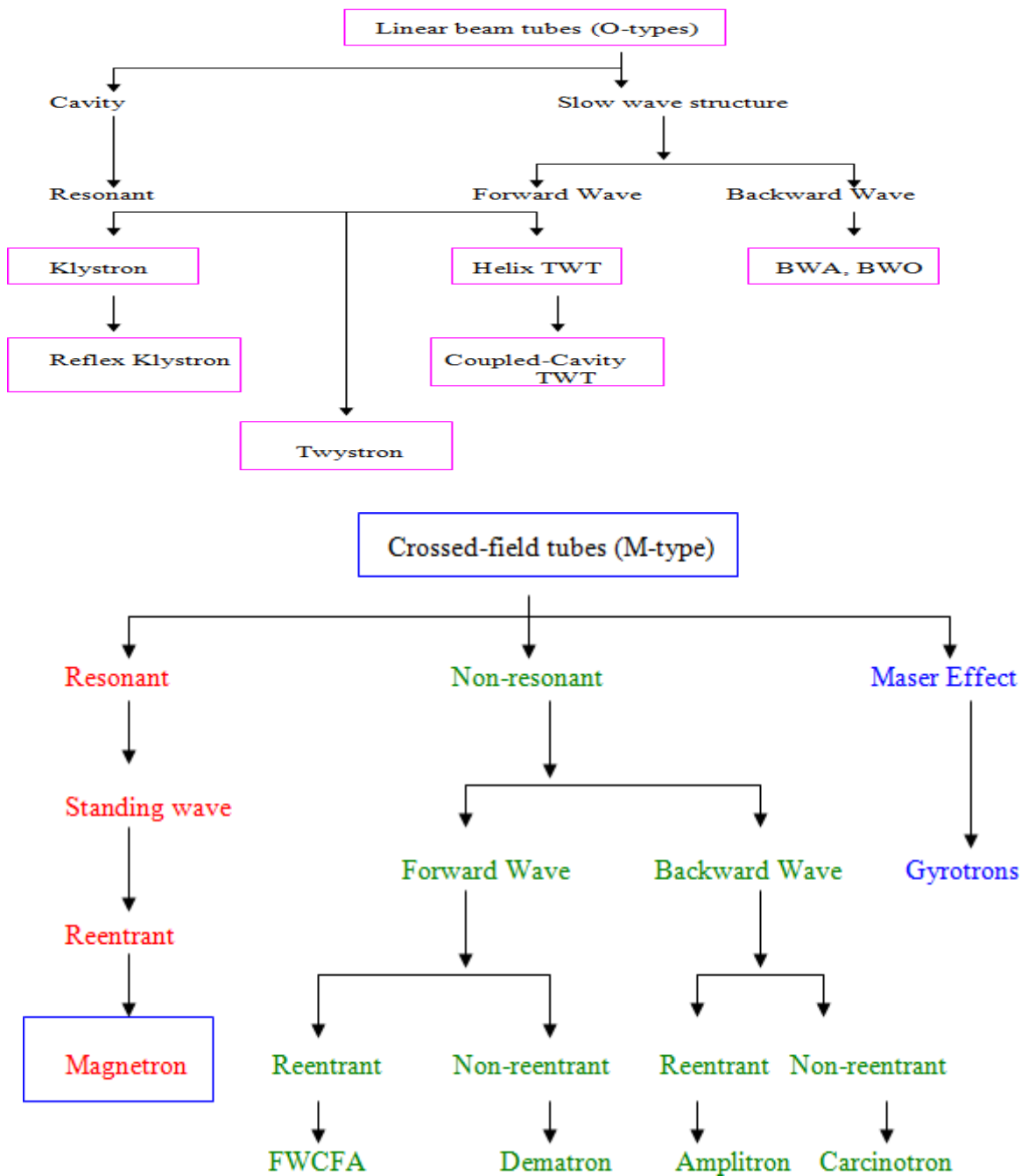
6) Dielectric loss

Dielectric material is generally different silicon plastic encapsulation materials used in microwave devices. At higher frequencies the losses due to these materials are also prominent.

Microwave Tubes:

1. Linear beam tubes (O-type)-Dc magnetic field is in parallel with the dc electric field.

2. Crossed-field tubes (M-type)-Dc electric field and the dc magnetic field are perpendicular to each other.



Cavity Klystrons

In microwave region, performs the functions of generates, receives and amplifies signals

Configurations:

1. Reflex – low power microwave oscillator
2. Multicavity – low power microwave amplifier

a) Reflex Klystron

- Has a reflector and one cavity as a resonator
- Reflex action of electron beam

Performance:

- Frequency range: 2-200 GHz
- BW: ± 30 MHz for $V_R: \pm 10$ V
- Power o/p: 10mW – 2.5W
- used as microwave source in lab, microwave transmitter
- frequency modulation and amplitude modulation

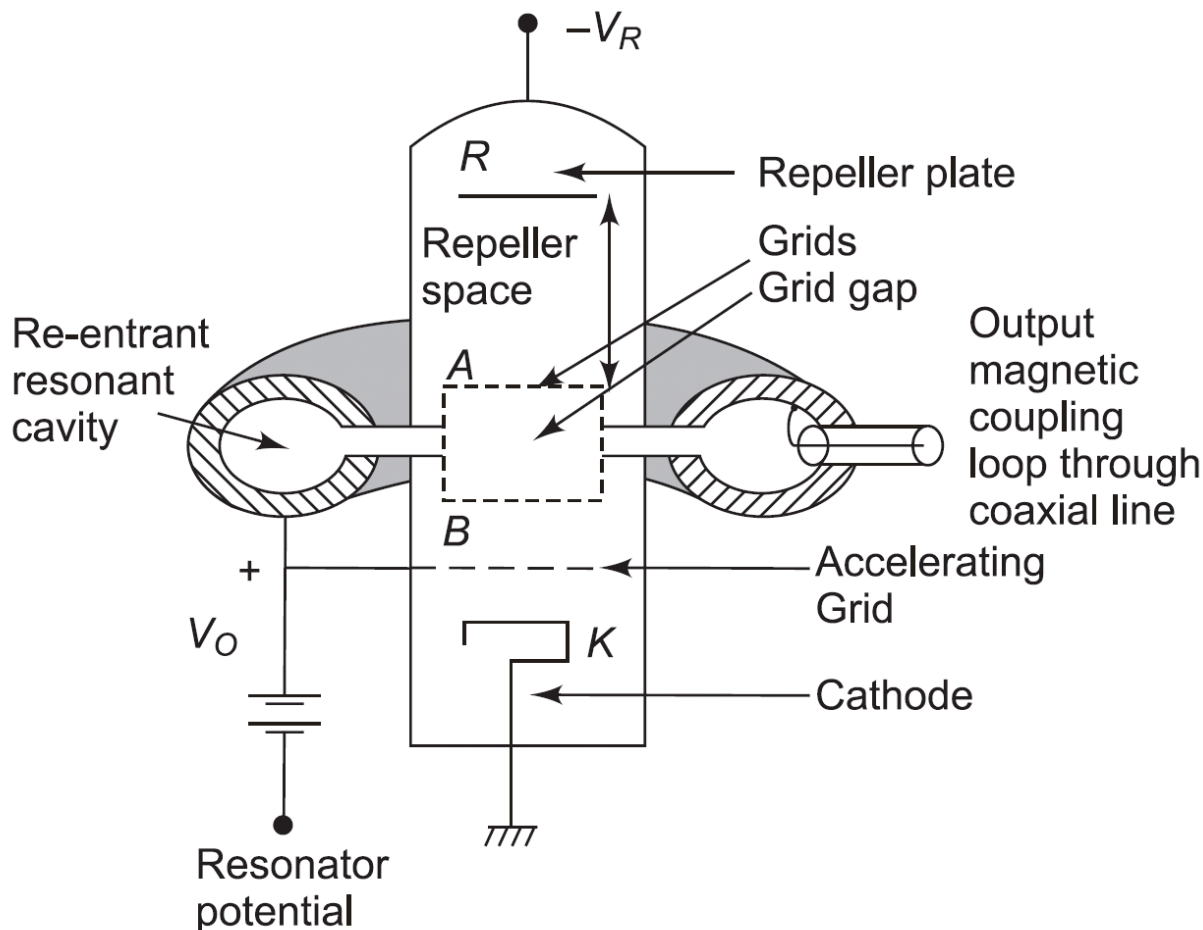


Fig. 9.3 *Functional diagram of a reflex klystron*

Mechanism of oscillation

- _ The electron passing through the cavity gap
- experience the RF field

_ velocity modulated

a: Electrons which encountered the positive half cycle of the RF field in the cavity gap will be accelerated

b: Electrons which encountered zero RF field will pass with unchanged original velocity

c: Electrons which encountered the negative half cycle will be retarded and entering the repeller space.

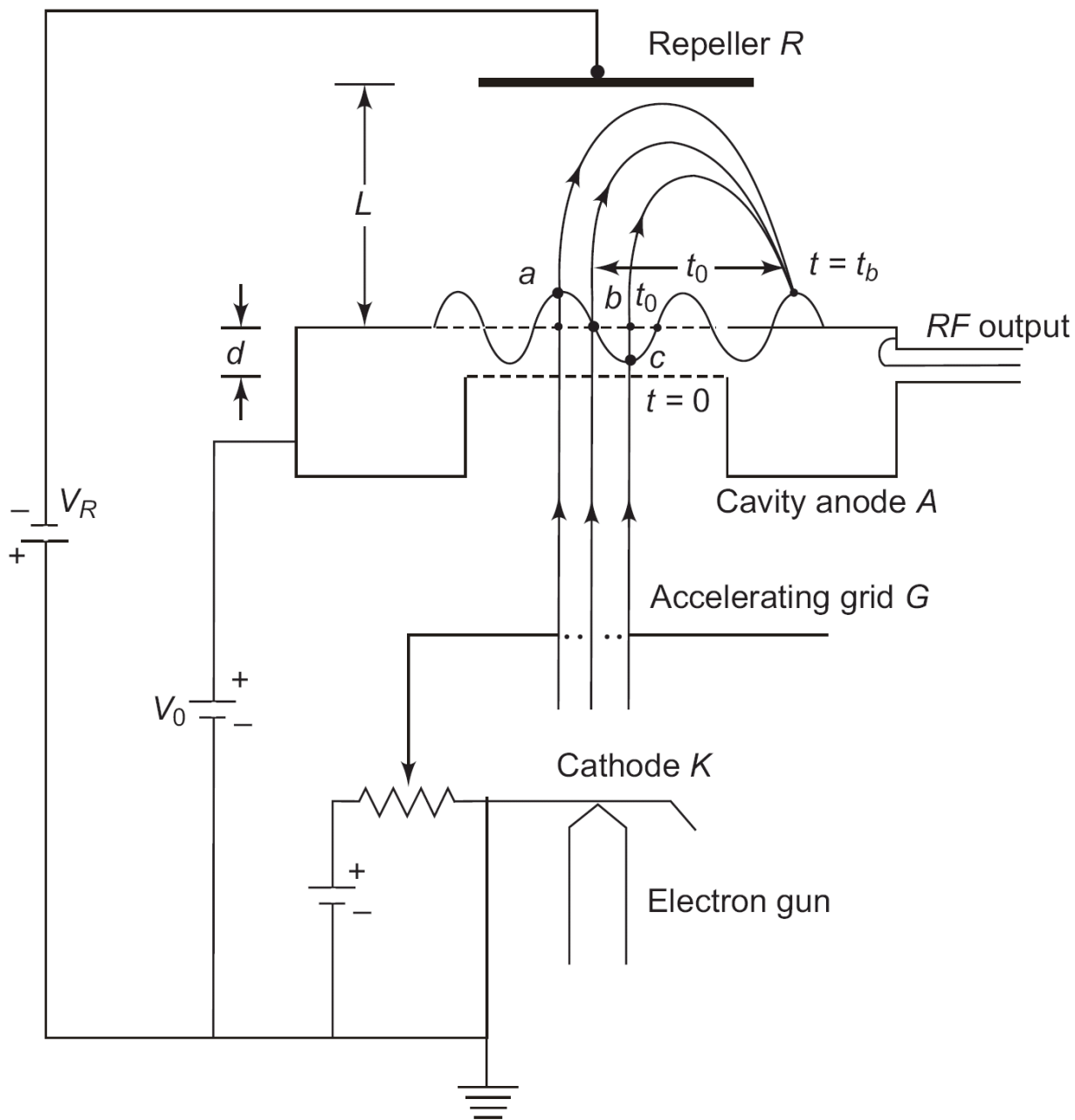


Fig. 9.4 *Reflex klystron operation*

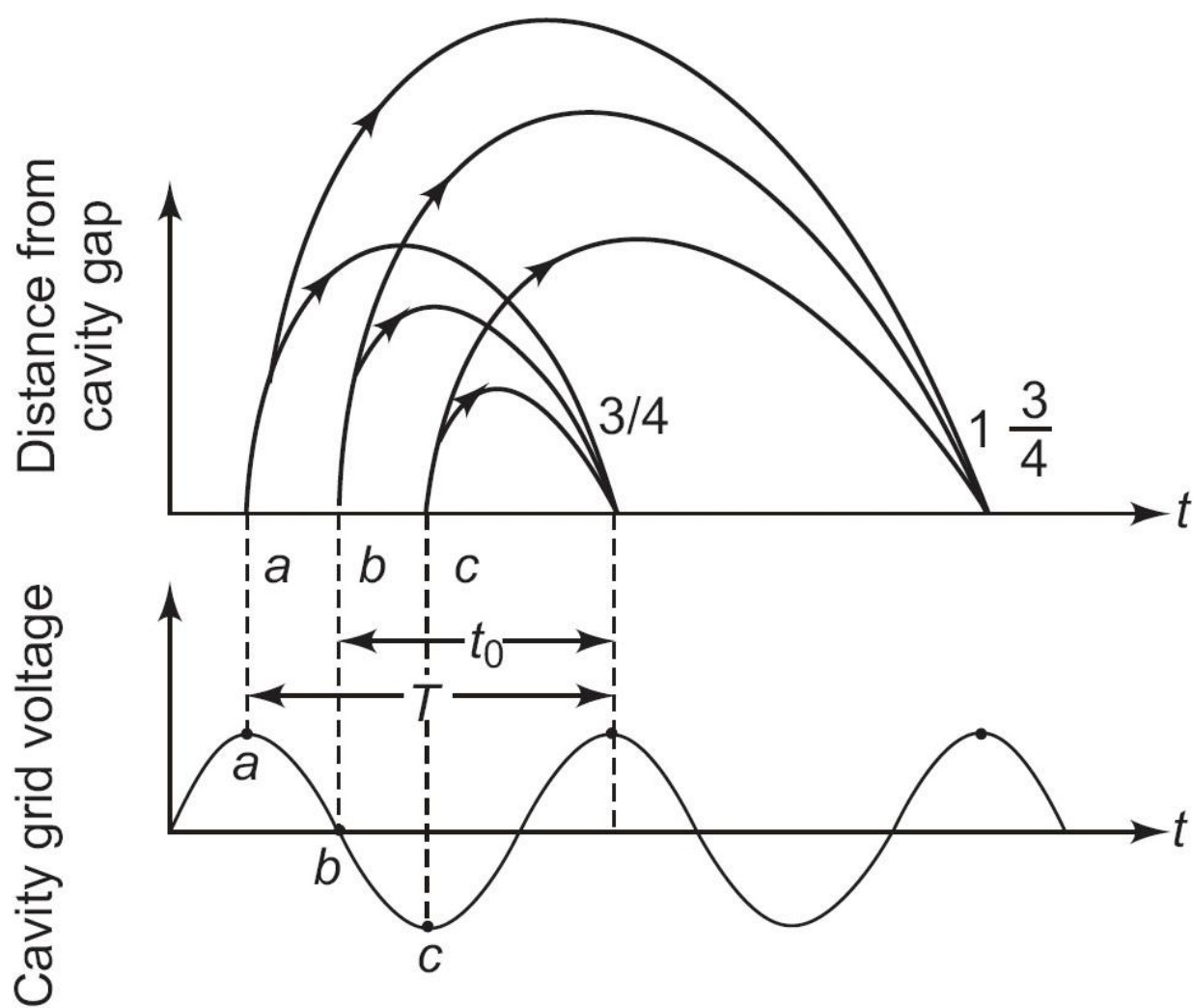
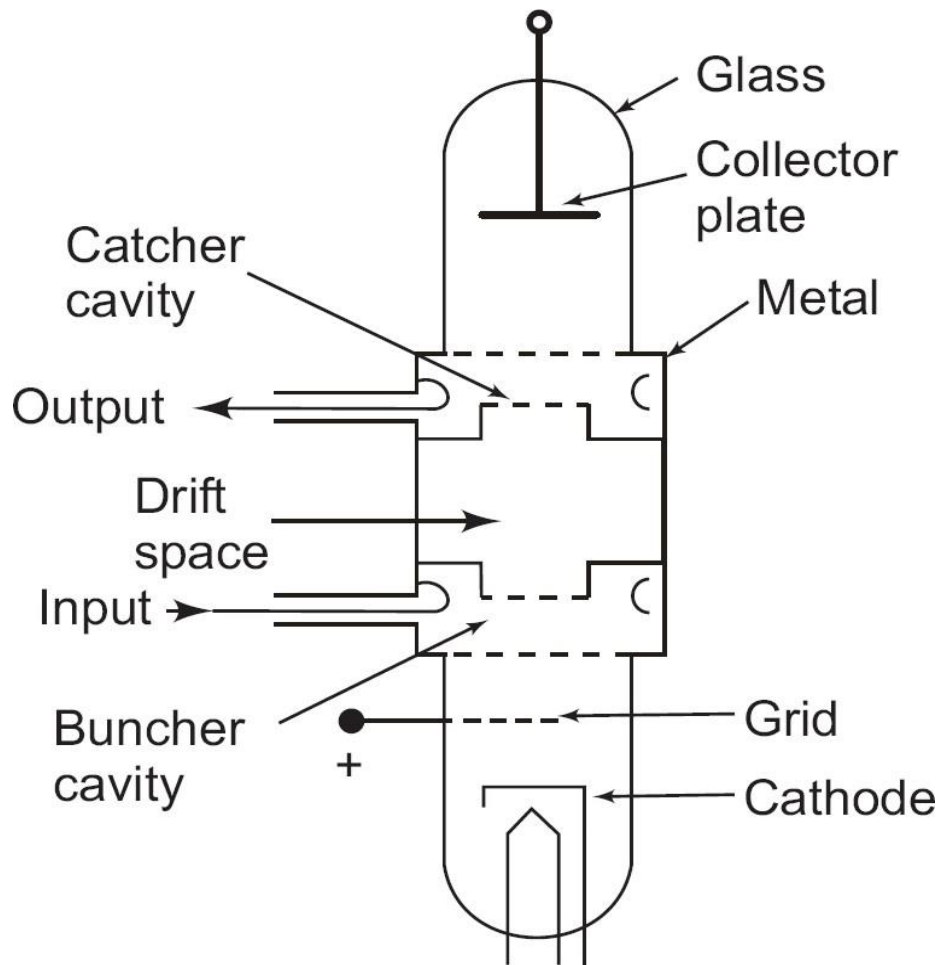


Fig. 9.5 *Reflex klystron modes*

Multicavity Klystron

Two cavity Klystron Amplifier

- _ Assumption for RF amplification
- _ Transit time in the cavity gap is very small compared to the period of the input RF signal cycle
- _ Input RF signal amplitude is very small compared to the dc beam voltage
- _ The cathode, anode, cavity grids and collectors are all parallel
- _ No space charge or debunching take place at the bunch point
- _ The RF fields are totally confined in the cavity gaps, zero in the drift space
- _ Electrons leave the cathode with zero initial velocity



(a)

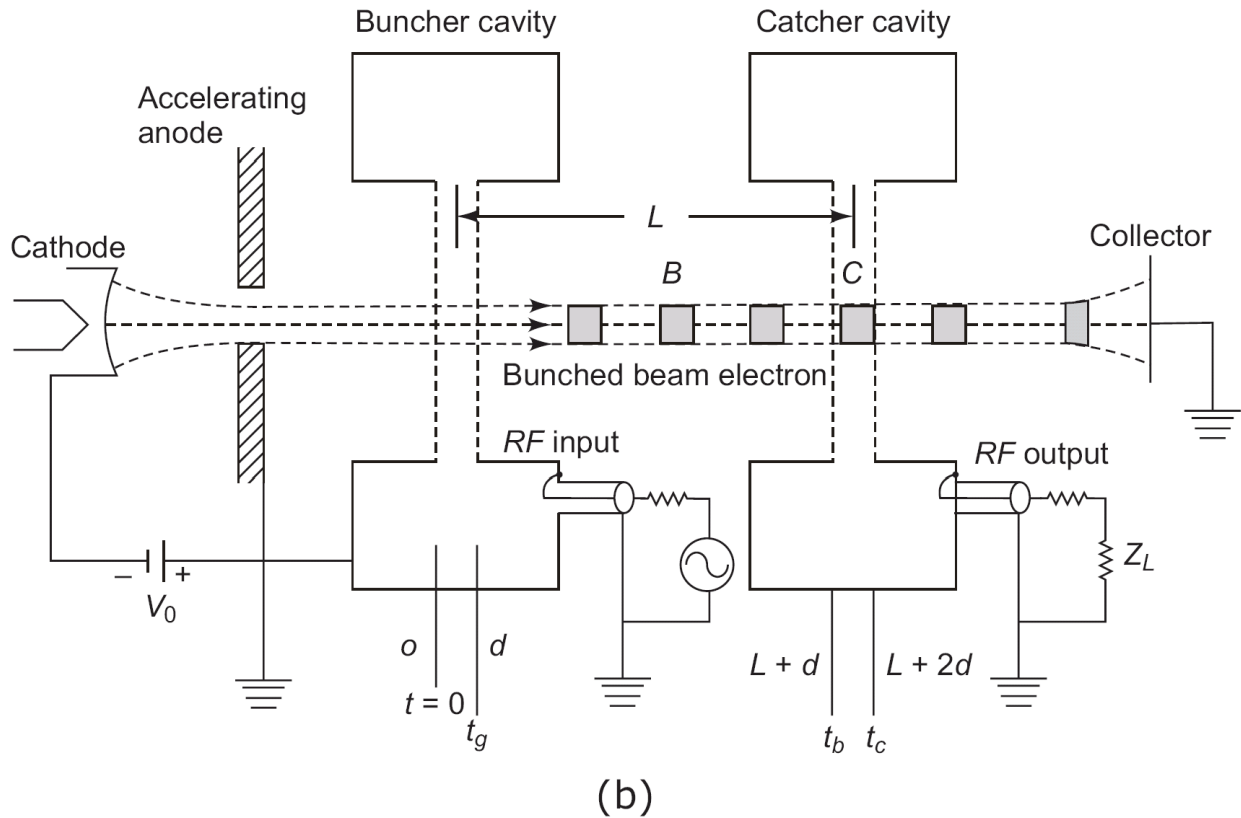


Fig. 9.12 Two-cavity klystron amplifier: (a) Schematic diagram
(b) Functional diagram

Traveling wave tube (TWT)

Travelling Wave Tube Amplifier:

- _ High gain > 40 dB
- _ Low NF < 10 dB
- _ Wide Band > Octave
- _ Frequency range: 0.3 – 50 GHz

- _ Contains electron gun, RF interaction circuit, electron beam focusing magnet, collector
- _ Amplify a weak RF input signal many thousands of times

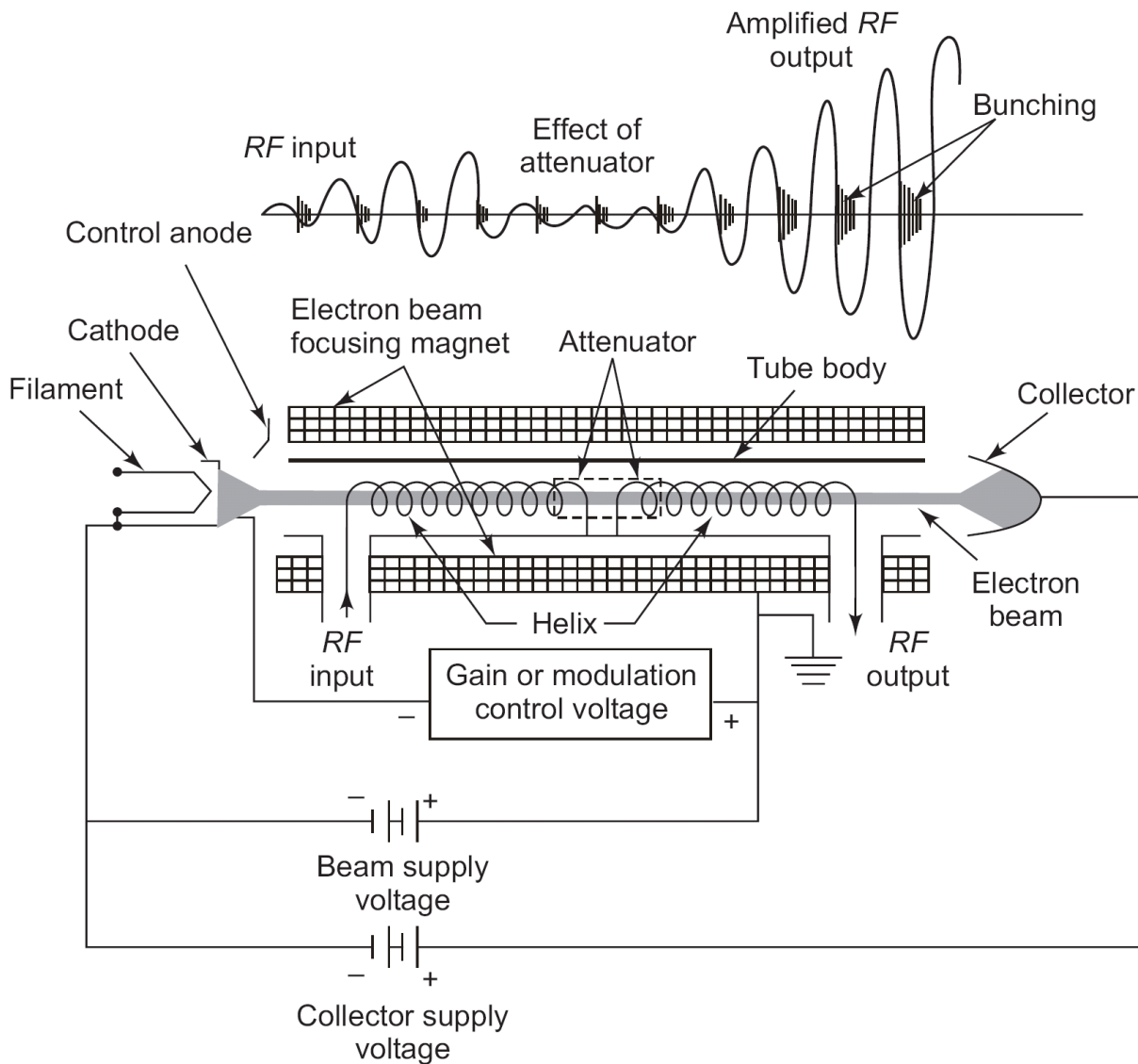


Fig. 9.18 *TWT amplifier tube and circuit*

a) Electron gun

_ To get as much electron current flowing into as small a region as possible without distortion or fuzzy edges

Sources of electrons for the beam- 6 elements:

- gun shells
- heater
- cathode
- control grid
- focus electrode
- anode

b) RF interaction circuit

- _ Interaction structures : helix, ring bar, ringloop, coupled cavity
- _ RF circuit – complex trade off analysis, based on many interlocking parameters
- _ Low power level : helix
- _ Medium power level : ring loop, ring bar
- _ Power level & frequency increased: RF losses on the circuit become more appreciable.

c) Electron beam focusing

- _ A magnetic field – to hold the electron beam together as it travels through the interaction structure of the tube
- _ The beam tends to disperse or spread out as a result of the natural repulsive forces between electrons.
- _ Methods of magnetic focusing
 - _ Solenoid magnetic structure
 - _ Permanent magnet
 - _ Periodic permanent magnet (PPM)
 - _ Radial magnet PPM

d) The collector

- _ To dissipate the electrons in the form of heat as they emerge from the slow wave structure
- _ Accomplished by thermal conduction to a colder outside surface – the heat is absorbed by circulated air or a liquid

1. Gain compression

- _ the amount of gain decrease from the small signal condition (normally 6dB)

2. Beam Voltage

- _ the voltage between the cathode and the RF structure

3. Synchronous Voltage

- _ the beam voltage necessary to obtain the greatest interaction between the electrons in the electron beam and the RF wave on the circuit

4. Gain

- _ the ratio of RF output power to RF input power (dB)

5. Phase Characteristic

- _ Phase shift – the phase of output signal relative to the input signal
- _ Phase sensitivity – the rate of phase change with a specific operating parameter

UNIT- IV

TRANSFER ELECTRON DEVICES

INTRODUCTION:

The application of two-terminal semiconductor devices at microwave frequencies has been increased usage during the past decades. The CW, average, and peak power outputs of these devices at higher microwave frequencies are much larger than those obtainable with the best power transistor. The common characteristic of all active two-terminal solid-state devices is their negative resistance. The real part of their impedance is negative over a range of frequencies. In a positive resistance the current through the resistance and the voltage across it are in phase. The voltage drop across a positive resistance is positive and a power of $(I^2 R)$ is dissipated in the resistance.

In a negative resistance, however, the current and voltage are out of phase by 180° . The voltage drop across a negative resistance is negative, and a power of $(-I^2 R)$ is generated by the power supply associated with the negative resistance. In positive resistances absorb power (passive devices), whereas negative resistances generate power (active devices). In this chapter the transferred electron devices (TEDs) are analyzed.

The differences between microwave transistors and transferred electron devices (TEDs) are fundamental. Transistors operate with either junctions or gates, but TEDs are bulk devices having no junctions or gates. The majority of transistors are fabricated from elemental semiconductors, such as silicon or germanium, whereas TEDs are fabricated from compound semiconductors, such as gallium arsenide (GaAs), indium phosphide (InP), or cadmium telluride (CdTe). Transistors operate as "warm" electrons whose energy is not much greater than the thermal energy (0.026 eV at room temperature) of electrons in the semiconductors.

GUNN EFFECT DIODES – GaAs diode

Gunn effect are named after J. B. Gunn who in 1963 discovered a periodic fluctuation of current passing through the n- type gallium arsenide . when the applied voltage exceeded a certain critical value.

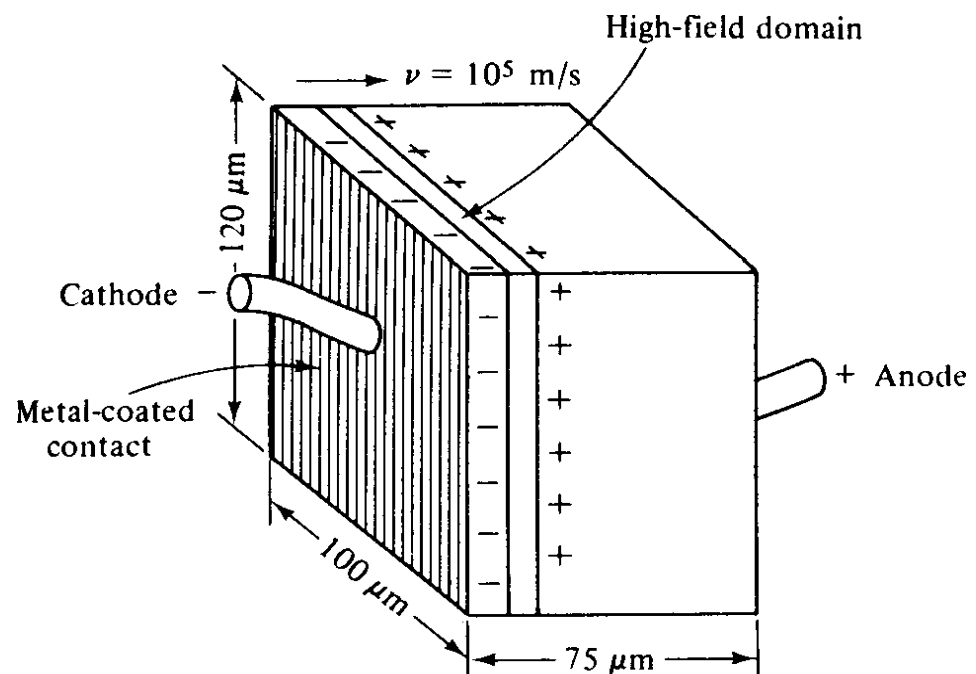
Shockley in 1954 suggested that the two terminal negative resistance devices using semiconductors had advantages over transistors at high frequencies.

In 1961 , Ridley and Watkins described a new method for obtaining negative differential mobility in semiconductors. The principle involved is to heat carriers in a light mass , low mobility , higher energy sub band when they have a high temperature.

Finally Kroemer stated that the origin of the negative differential mobility is Ridley Watkins Hilsum's mechanism of electron transfer into the valleys that occur in conduction bands.

Gunn effect:

The below figure shows the diagram of a uniform n-type GaAs diode with ohmic contacts at the end surfaces. Gunn stated that “ Above some critical voltage , corresponding to an electric field of 2000 to 4000 Volts/cm, the current in every specimen became a fluctuating function of time.



Gunn Diodes

Single piece of GaAs or Inp and contains no junctions Exhibits

negative differential resistance

Applications:

low-noise local oscillators for mixers (2 to 140 GHz). Low-power
transmitters and wide band tunable sources

Continuous-wave (CW) power levels of up to several hundred mill watts can be obtained in the X-, Ku-, and Ka-bands. A power output of 30 mW can be achieved from commercially available devices at 94 GHz.

Higher power can be achieved by combining several devices in a power combiner.

Gunn oscillators exhibit very low dc-to-RF efficiency of 1 to 4%.

Gunn also discovered that the threshold electric field E_{th} varied with the length and type of material. He developed an elaborate capacitive probe for plotting the electric field distribution within a specimen of n-type GaAs of length $L = 210 \mu\text{m}$ and cross-sectional area $3.5 \times 10^{-3} \text{ cm}^2$ with a low-field resistance of 16Ω .

Current instabilities occurred at specimen voltages above 59 V, which means that the threshold field is

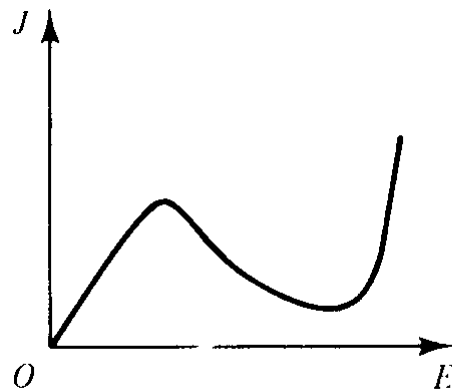
$$E_{th} = \frac{V}{L} = \frac{59}{210 \times 10^{-6} \times 10^2} = 2810 \text{ volts/cm}$$

RIDLEY WATKINS AND HILSUM THEORY:

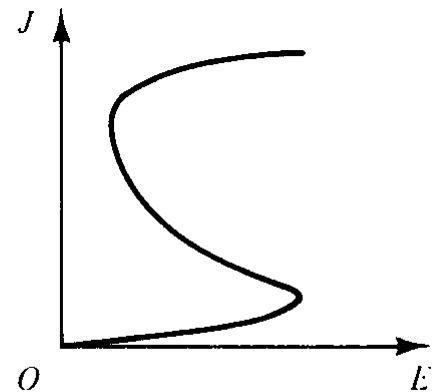
Many explanations have been offered for the Gunn effect. In 1964 Kroemer [6] suggested that Gunn's observations were in complete agreement with the Ridley- Watkins-Hilsum (RWH) theory.

Differential Negative Resistance:

The fundamental concept of the Ridley-Watkins-Hilsum (RWH) theory is the differential negative resistance developed in a bulk solid-state III-V compound when either a voltage (or electric field) or a current is applied to the terminals of the sample. There are two modes of negative-resistance devices: voltage- controlled and current controlled Modes.

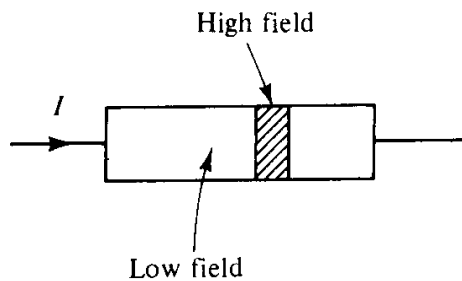


(a) Voltage-controlled mode

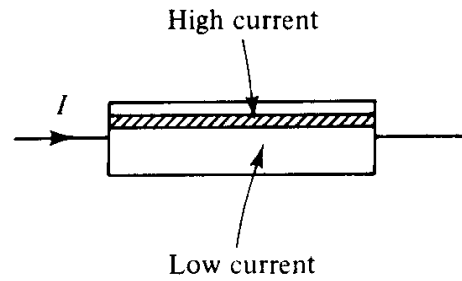


(b) Current-controlled mode

In the voltage-controlled mode the current density can be multivalued, whereas in the current-controlled mode the voltage can be multivalued. The major effect of the appearance of a differential negative-resistance region in the current density field curve is to render the sample electrically unstable. As a result, the initially homogeneous sample becomes electrically heterogeneous in an attempt to reach stability. In the voltage-controlled negative-resistance mode high-field domains are formed, separating two low-field regions. The interfaces separating low and high-field domains lie along equi potentials; thus they are in planes perpendicular to the current direction.



(a) High-field domain

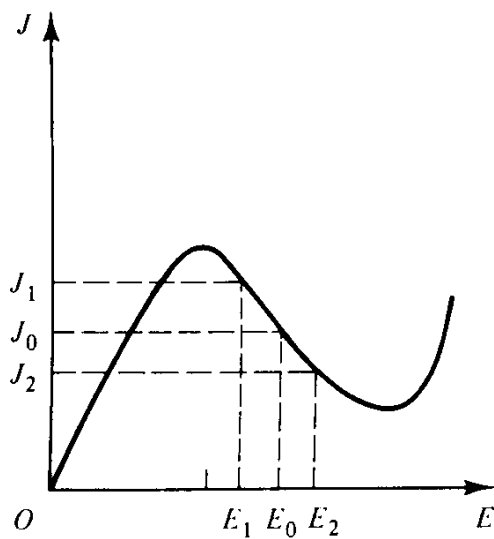


(b) High-current filament

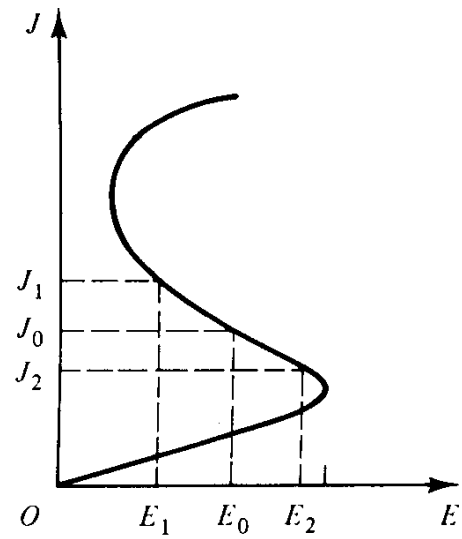
Expressed mathematically, the negative resistance of the sample at a particular region is

$$\frac{dI}{dV} = \frac{dJ}{dE} = \text{negative resistance}$$

If an electric field E_0 (or voltage V_0) is applied to the sample, for example, the current density J_0 is generated. As the applied field (or voltage) is increased to E_1 (or V_2), the current density is decreased to J_2 . When the field (or voltage) is decreased to E_2 (or V_1), the current density is increased to J_1 . These phenomena of the voltage controlled negative resistance are shown in Fig. 7-2-3(a). Similarly, for the current controlled mode, the negative-resistance profile is as shown below.



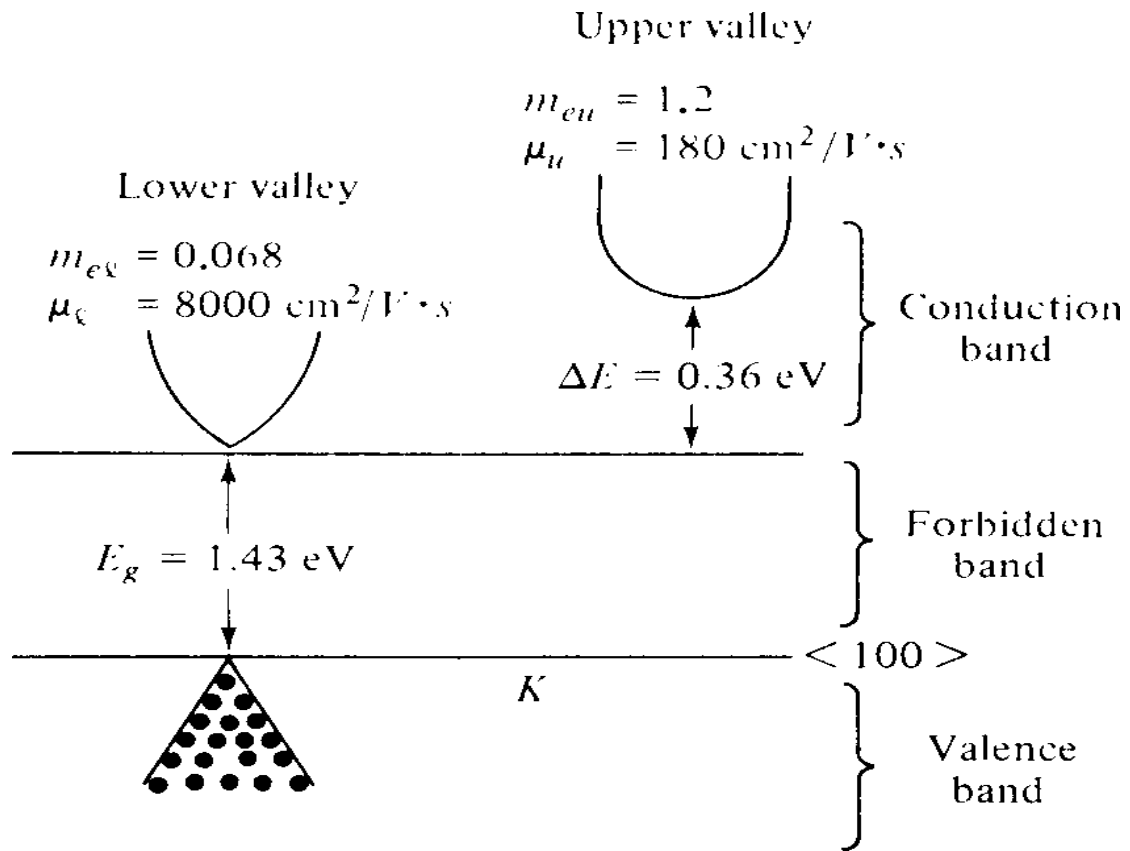
(a) Voltage-controlled mode



(b) Current-controlled mode

TWO VALLEY MODEL THEORY:

Kroemer proposed a negative mass microwave amplifier in 1958 [IO] and 1959 [II]. According to the energy band theory of the n -type GaAs, a high-mobility lower valley is separated by an energy of 0.36 eV from a low-mobility upper valley

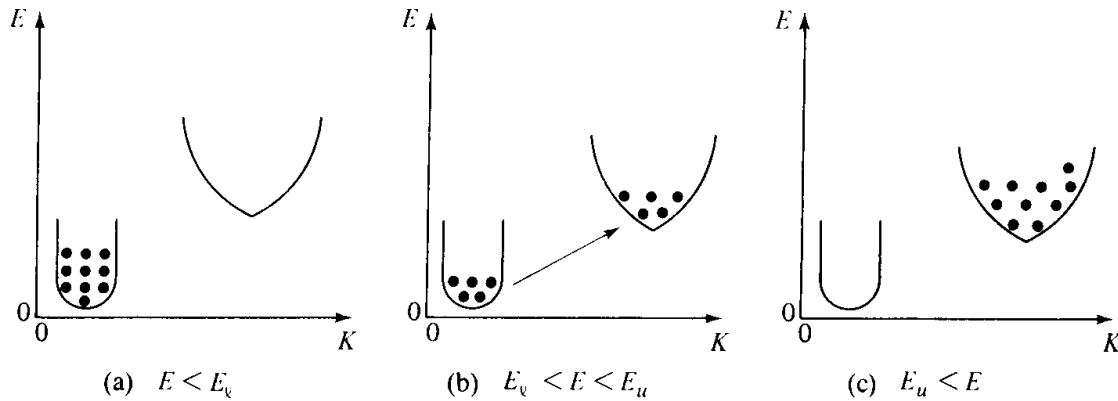


Electron densities in the lower and upper valleys remain the same under an Equilibrium condition. When the applied electric field is lower than the electric field of the lower valley ($E < E_{\ell}$), no electrons will transfer to the upper valley.

When the applied electric field is higher than that of the lower valley and lower than that of the upper valley ($E_{\ell} < E < E_u$), electrons will begin to transfer to the upper valley.

when the applied electric field is higher than that of the upper valley ($E_u < E$), all electrons will transfer to the upper valley.

When a sufficiently high field E is applied to the specimen, electrons are accelerated and their effective temperature rises above the lattice temperature also increases. Thus electron density n and T_e are both functions of electric field E .



Transfer of electron densities.

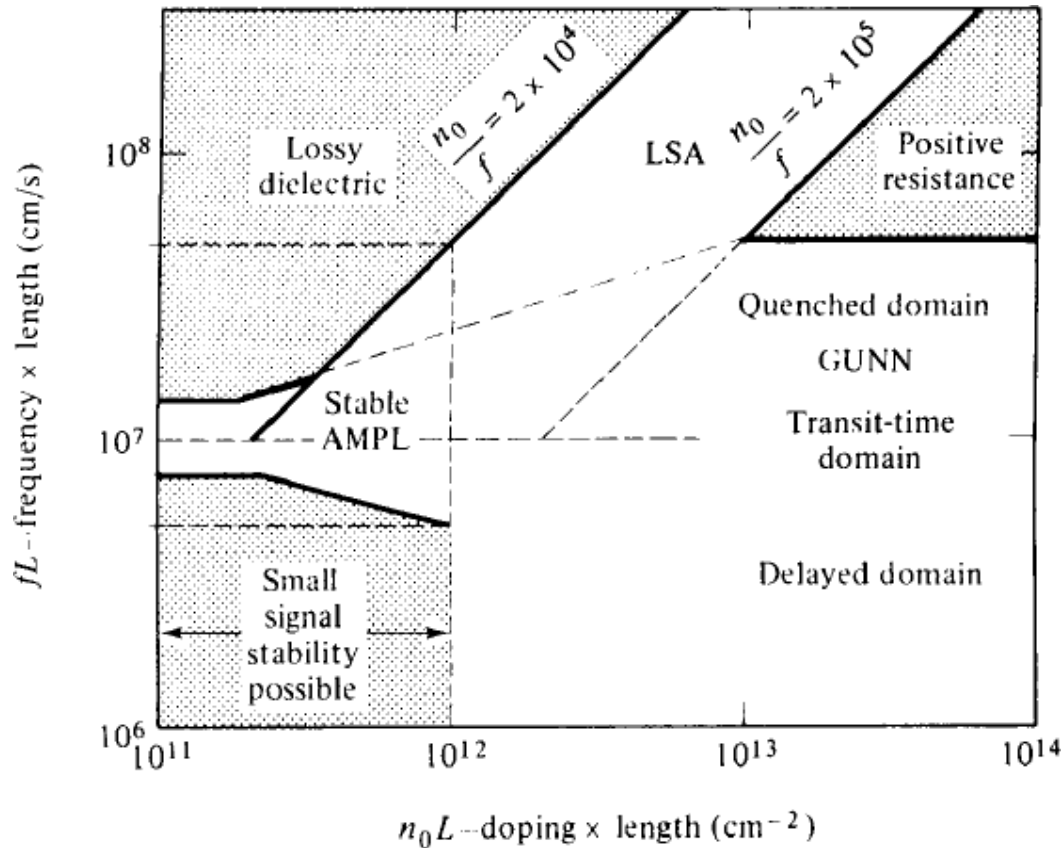
MODES OF OPERATION OF GUNN DIODE:

A Gunn diode can operate in four modes:

1. Gunn oscillation mode
2. stable amplification mode
3. LSA oscillation mode
4. Bias circuit oscillation mode

Gunn oscillation mode: This mode is defined in the region where the product of frequency multiplied by length is about 10^7 cm/s and the product of doping multiplied by length is greater than $10^{12}/\text{cm}^2$. In this region the device is unstable because of the cyclic formation of either the accumulation layer or the high field domain.

When the device is operated is a relatively high Q cavity and coupled properly to the load, the domain I quenched or delayed before nucleating.

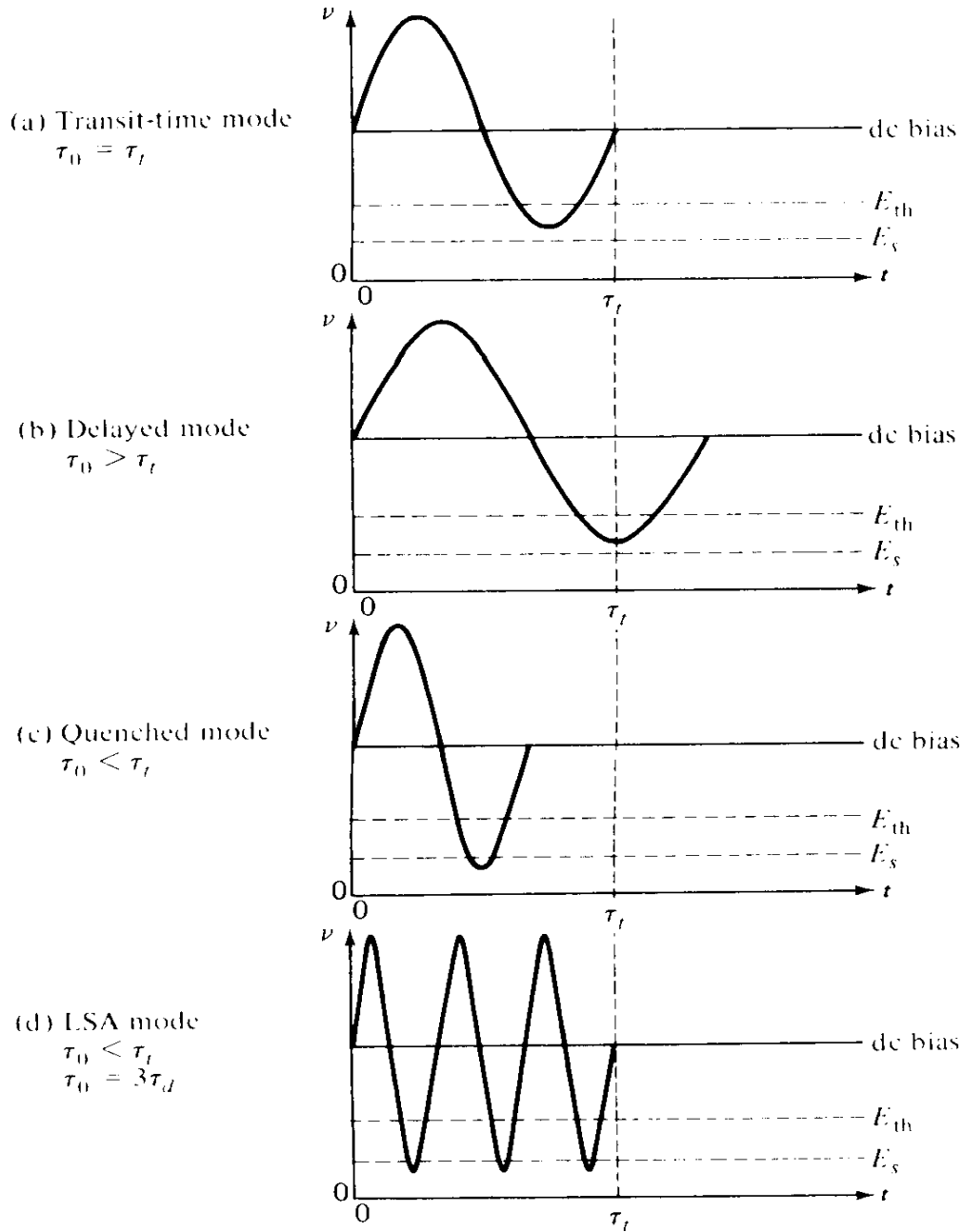


2. Stable amplification mode: This mode is defined in the region where the product of frequency times length is about 10^7 *cmls* and the product of doping times length is between 10^{11} and $10^{12}/\text{cm}^2$

3. LSA oscillation mode: This mode is defined in the region where the product of frequency times length is above 10^7 *cmls* and the quotient of doping divided by frequency is between 2×10^4 and 2×10^5 .

4. Bias-circuit oscillation mode: This mode occurs only when there is either Gunn or LSA oscillation, and it is usually at the region where the product of frequency times length is too small to appear in the figure. When a bulk diode is biased to threshold, the average current suddenly drops as Gunn oscillation begins.

The drop in current at the threshold can lead to oscillations in the bias circuit that are typically 1 kHz to 100 MHz .



Delayed domain mode ($106 \text{ cm/s} < fL < 107 \text{ cm/s}$). When the transit time is Chosen so that the domain is collected while $E < E_{th}$ as shown in Fig. 7-3-4(b), a new domain cannot form until the field rises above threshold again. In this case, the oscillation period is greater than the transit time- that is, $T_o > T_t$. This delayed mode is also called *inhibited mode*. The efficiency of this mode is

about 20%.

Quenched domain mode ($fL > 2 \times 10^7$ cm/s).

If the bias field drops below the sustaining field E_s during the negative half-cycle as shown, the domain collapses before it reaches the anode. When the bias field swings back above threshold, a new domain is nucleated and the process repeats. Therefore the oscillations occur at the frequency of the resonant circuit rather than at the transit-time frequency. It has been found that the resonant frequency of the circuit is several times the transit-time frequency, since one dipole does not have enough time to readjust and absorb the voltage of the other dipoles. Theoretically, the efficiency of quenched domain oscillators can reach 13%

LSA MODE

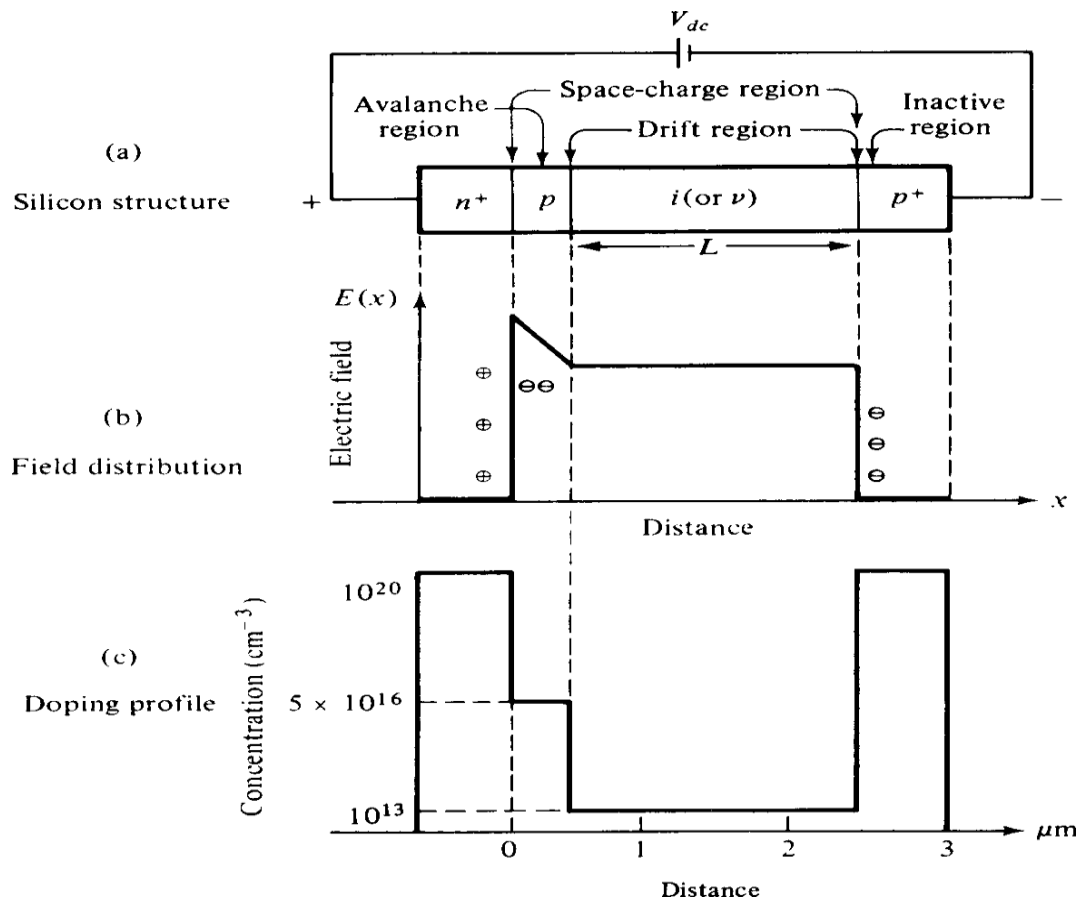
When the frequency is very high, the domains do not have sufficient time to form while the field is above threshold. As a result, most of the domains are maintained in the negative conductance state during a large fraction of the voltage cycle. Any accumulation of electrons near the cathode has time to collapse while the signal is below threshold. Thus the LSA mode is the simplest mode of operation.

AVALANCHE TRANSIT TIME DEVICES:

READ DIODE:

Read diode was the first proposed avalanche diode. The basic operating principles of IMPATT diode can be easily understood by first understanding the operation of read diode.

The basic read diode consists of four layers namely $n^+ p i p^+$ layers. The plus superscript refers to very high doping levels and 'i' denotes intrinsic layer. A large reverse bias is applied across diode. The avalanche multiplication occurs in the thin "p" region which is also called the high field region or avalanche region.



The holes generated during the avalanche process drift through the intrinsic region while moving towards p+ contact. The region between n+ p junction and the i-p+ junction is known as space charge region.

When this diode is reverse biased and placed inside an inductive microwave cavity microwave oscillations are produced due to the resonant action of the capacitive impedance of the diode and cavity inductance. The dc bias power is converted into microwave power by that read diode oscillator.

Avalanche multiplication occurs when the applied reverse bias voltage is greater then the breakdown voltage so that the space charge region extends from n+p junction through the p and I regions, to the i to p+ junction.

IMPATT DIODE:

Impatt diodes are manufactured having different forms such as n+pip+, p+nin+, p+nn+ abrupt junction and p+ i n+ diode configuration. The material used for manufacture of these modes are either Germanium, Silicon, Gallium Arsenide (GaAs) or Indium Phosphide (In P).

Out of these materials, highest efficiency, higher operating frequency and lower noise is obtained with GaAs. But the disadvantage with GaAs is complex fabrication process and hence higher cost. The

figure below shows a reverse biased n+ pi p+ diode with electric field variation, doping concentration versus distance plot, the microwave voltage swing and the current variation.

PRINICPLE OF OPERATION:

When a reverse bias voltage exceeding the breakdown voltage is applied, a high electric field appears across the n+ p junction. This high field intensity imparts sufficient energy to the valence electrons to raise themselves into the conduction band. This results avalanche multiplication of hole-electron pairs. With suitable doping profile design, it is possible to make electric field to have a very sharp peak in the close vicinity of the junction resulting in "impact avalanche multiplication". This is a cumulative process resulting in rapid increase of carrier density. To prevent the diode from burning, a constant bias source is used to maintain average current at safe limit 10, The diode current is contributed by the conduction electrons which move to the n+ region and the associated holes which drift through the steady field and a.c. field. The diode ~wings into and out of avalanche conditions under the influence of that reverse bias steady field and the a.c. field.

Due to the drift *time* of holes being' small, carriers drift to the end contacts before the a.c. voltage swings the diode out of the avalanche Due to building up of oscillations, the a.c. field takes energy from the applied bias lid the oscillations at microwave frequencies are sustained across the diode. Due to this a.c. field, the hole current grows exponentially to a maximum and again decays exponentially to Zero.

During this hole drifting process, a constant electron current is induced in the external Circuit which starts flowing when hole current reaches its peak and continues for half cycle Corresponding to negative swing of the a.c. voltage as shown in figure Thus a 180 degrees Phase shift between the external current and a.c. microwave voltage provides a negative Resistance for sustained oscillations.

The resonator is usually tuned to this frequency so that the IMPATI diodes provide a High power continuous wave (CW) and pulsed microwave signals.

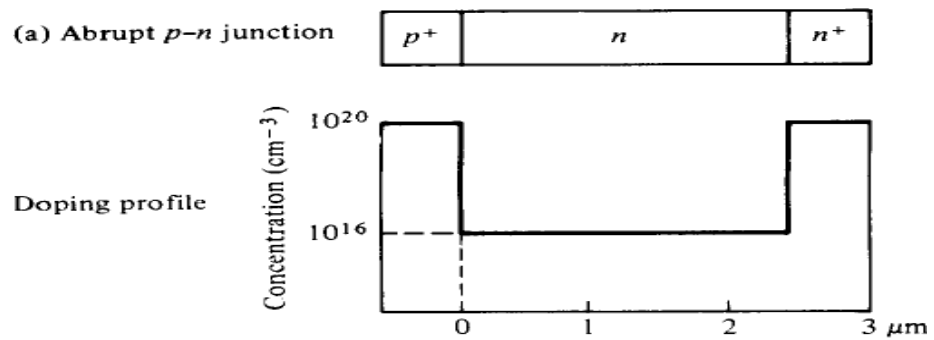
$$\theta = \omega\tau = \omega \frac{L}{v_d}$$

$$\omega_r \equiv \left(\frac{2\alpha' v_d I_0}{\epsilon_s A} \right)^{1/2}$$

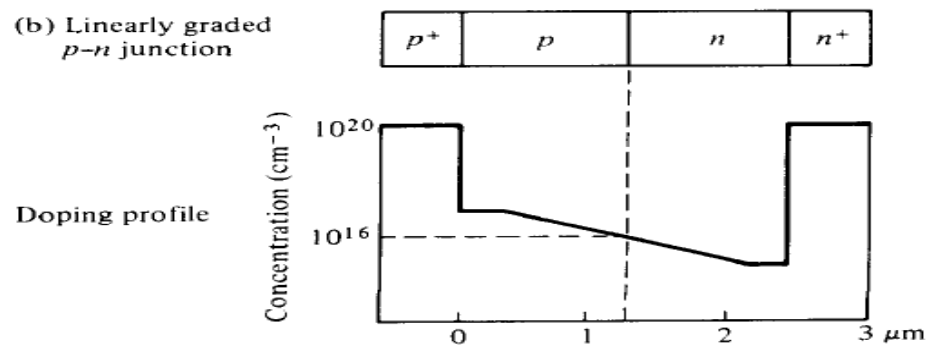
Applications of IMPATT Diodes

- (i) Used in the final power stage of solid state microwave transmitters for communication purpose.
- (ii) Used in the transmitter of TV system.
- (iii) Used in FDM/TDM systems.
- (iv) Used as a microwave source in laboratory for measurement purposes.

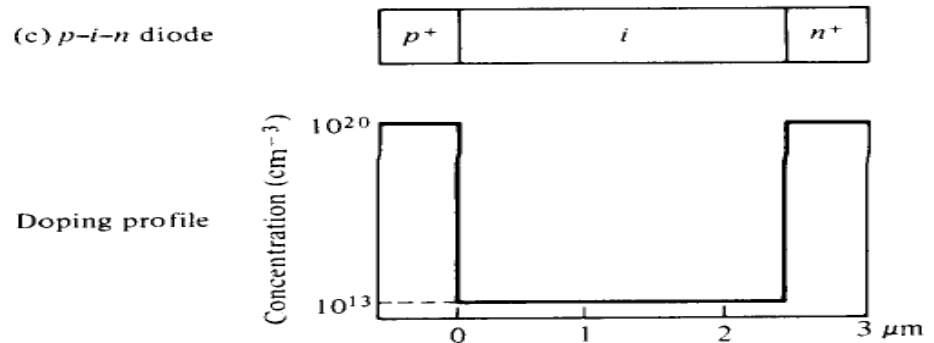
(a) Abrupt p - n junction



(b) Linearly graded p - n junction



(c) p - i - n diode



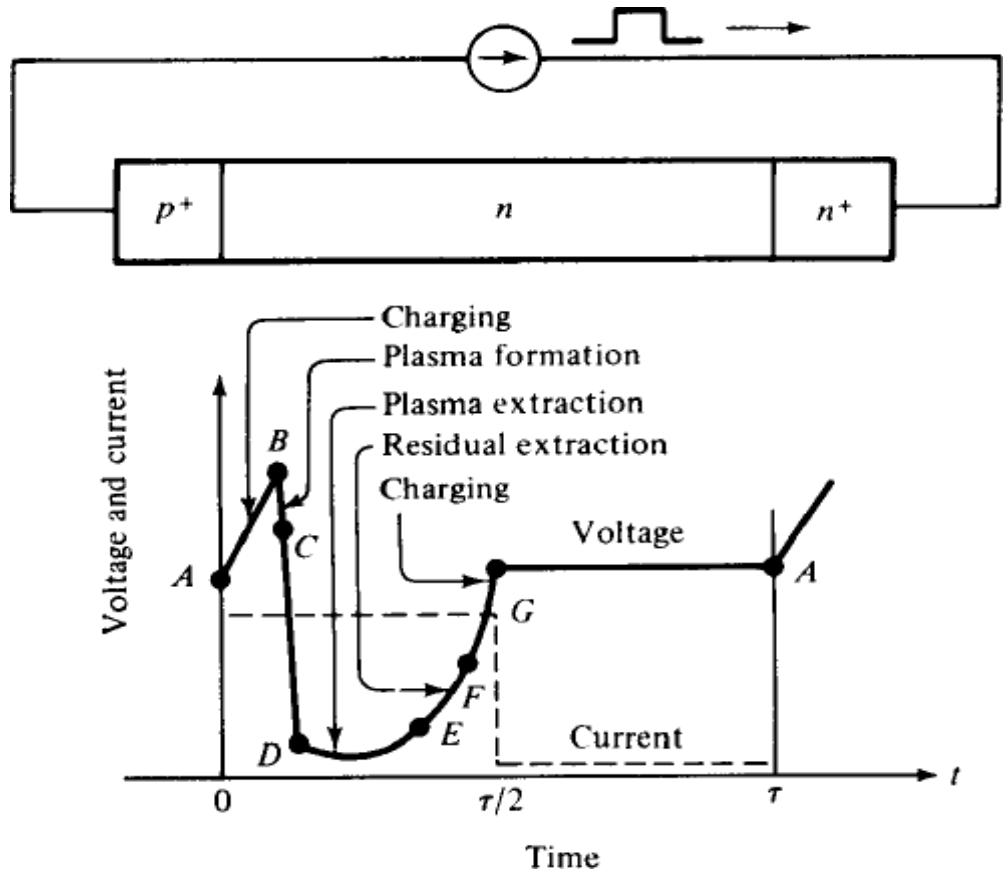
TRAPATT DIODE:

Silicon is usually used for the manufacture of TRAPATT diodes and have a configuration of $p^+ n n^+$ as shown. The p-N junction is reverse biased beyond the breakdown region, so that the current density is larger. This decreases the electric field in the space charge region and increases the carrier transit time. Due to this, the frequency of operation gets lowered to less than 10 GHz. But the efficiency gets increased due to low power dissipation.

Inside a co-axial resonator, the TRAPATT diode is normally mounted at a point where maximum RF voltage swing is obtained. When the combined dc bias and RF voltage exceeds breakdown voltage, avalanche occurs and a plasma of holes and electrons are generated which gets trapped. When the external circuit current flows, the voltage rises and the trapped plasma gets released producing current pulse across the drift space. The total transit time is the sum of the drift time and the delay introduced by the release of the trapped plasma. Due to this longer transit time, the operating frequency is limited to 10 GHz. Because the current pulse is associated with low voltage, the power dissipation is low resulting in higher efficiency.

The disadvantages of TRAPATT are high noise figure and generation of strong harmonics due to short duration of the current pulse.

TRAPATT diode finds application in S-band pulsed transmitters for pulsed array radar systems.



The electric field is expressed as

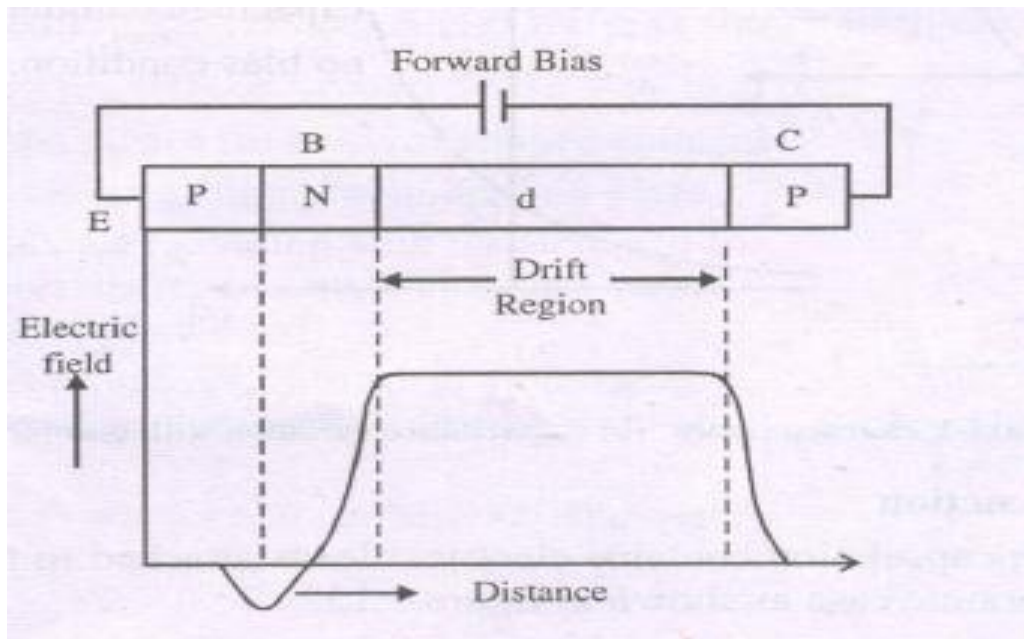
$$E(x, t) = E_m - \frac{qN_A}{\epsilon_s}x + \frac{Jt}{\epsilon_s}$$

BARITT DIODE (Barrier injection transmit time devices):

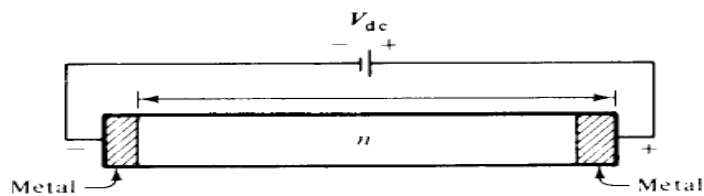
BARITT devices are an improved version of IMPATT devices. IMPATT devices employ impact ionization techniques which is too noisy. Hence in order to achieve low noise figures, impact ionization is avoided in BARRITT devices. The minority injection is provided by punch-through of the intermediate region (depletion region). The process is basically of lower noise than impact ionization responsible for current injection in an IMPATT. The negative resistance is obtained on account of the drift of the injected holes to the collector end of the aternal.

The construction of a BARITT device consisting of emitter, base, intermediate or drift or depleted region and collector. An essential requirement for the BARITT device is therefore that the intermediate drift region be entirely depleted to cause punch through to the emitter-base junction without causing avalanche breakdown of the base-

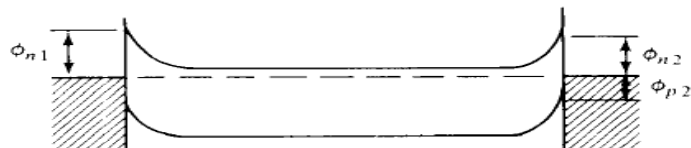
collector junction.



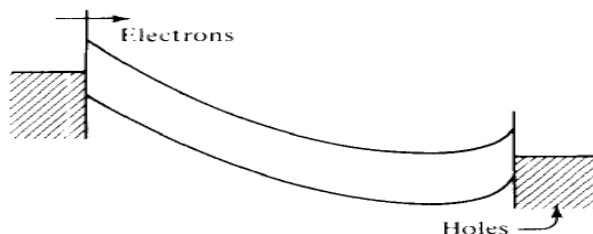
The parasitic should be kept as low as possible. The equivalent circuit depends on the type of encapsulation and mounting make. For many applications, there should be a large capacitance variation, small value of minimum capacitance and series resistance R_s . Operation is normally limited to $f/10$ [25 GHz for Si and 90 GHz for $GaAs$]. Frequency of operation beyond $(f/10)$ leads to increase in R , decrease in efficiency and increase in noise.



(a) M-n-M diode



(b) Energy band diagram in thermal equilibrium



(c) Energy band under bias condition

UNIT- V

ATTENUATORS:

In order to control power levels in a microwave system by partially absorbing the transmitted microwave signal, attenuators are employed. Resistive films (dielectric glass slab coated with aquadag) are used in the design of both fixed and variable attenuators. A co-axial fixed attenuator uses the dielectric lossy material inside the centre conductor of the co-axial line to absorb some of the centre conductor microwave power propagating through it dielectric rod decides the amount of attenuation introduced. The microwave power absorbed by the lossy material is dissipated as heat.

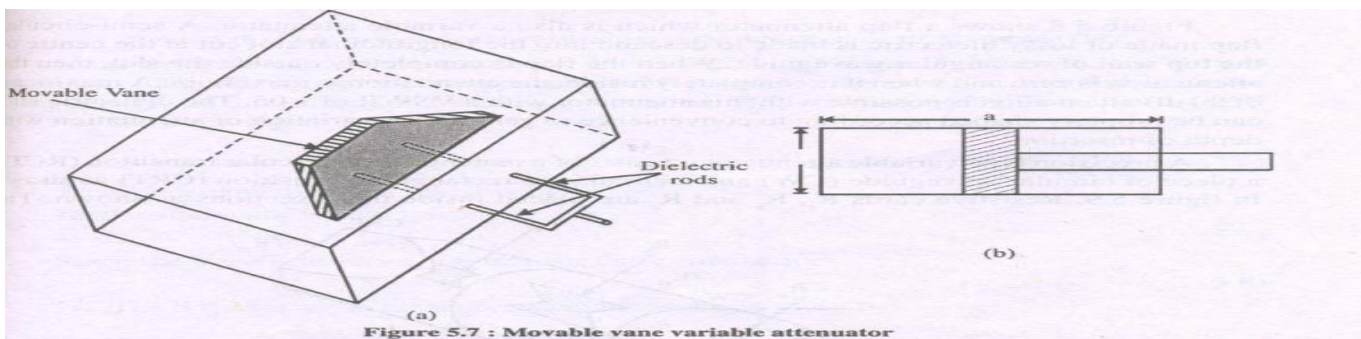
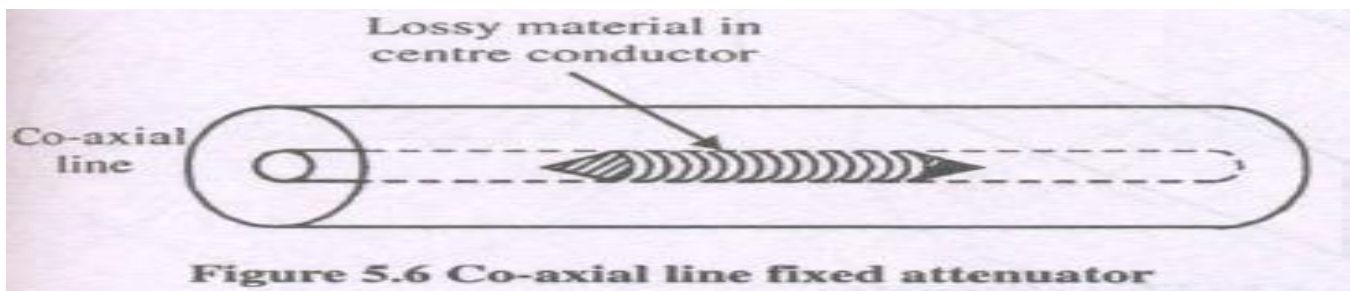
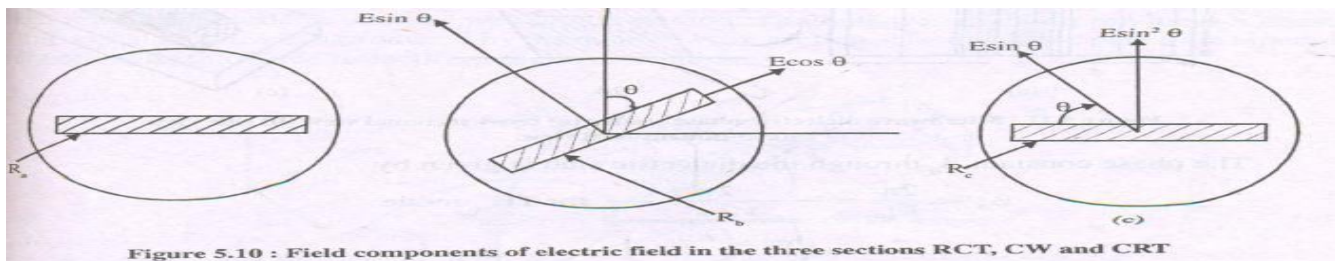
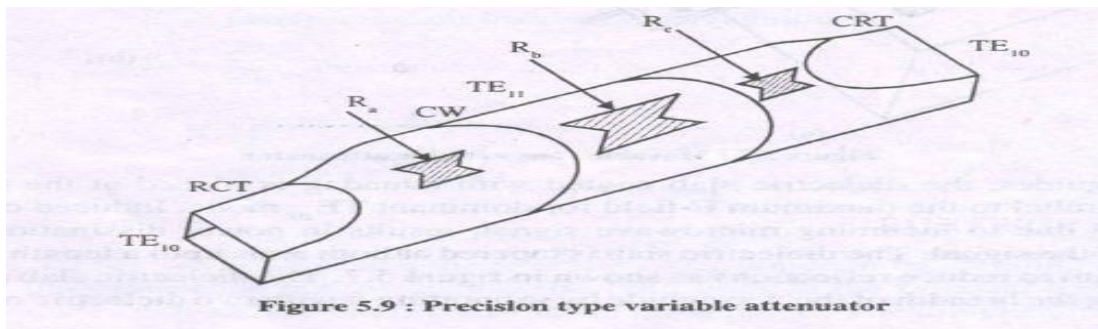


Figure 5.8 shows a flap attenuator which is also a variable attenuator. A semi- circular flap made of lossy dielectric is made to descend into the longitudinal slot cut at the centre of the top wall of

rectangular waveguide. When the flap is completely outside the slot, then the attenuation is zero and when it is completely

inside, the attenuation is maximum. A maximum direction of 90 dB attenuation is possible with this attenuator with a VSWR of 1.05. The dielectric slab can be properly shaped according to convenience to get a linear variation of attenuation within the depth of insertion.

A precision type variable attenuator consists of a rectangular to circular transition (ReT), a piece of circular waveguide (CW) and a circular-to-rectangular transition



PHASE SHIFTERS:

A microwave phase shifter is a two port device which produces a variable shift in phase of the incoming microwave signal. A lossless dielectric slab when placed inside the rectangular waveguide produces a phase shift.

PRECISION PHASE SHIFTER

The rotary type of precision phase shifter is shown in figure 5.12 which consists of a circular waveguide containing a lossless dielectric plate of length $2l$ called "half- wave section", a section of

rectangular-to-circular transition containing a lossless dielectric plate of length l , called "quarter-wave section", oriented at an angle of 45° to the broader wall of the rectangular waveguide and a circular-to-rectangular transition again containing a

lossless dielectric plate of same length l (quarter wave section) oriented at an angle 45° . The incident TEIO mode becomes TELL mode in circular waveguide section. The half-wave section produces a phase shift equal to twice that produced by the quarter wave section. The dielectric plates are tapered at both ends to reduce reflections due to discontinuity.

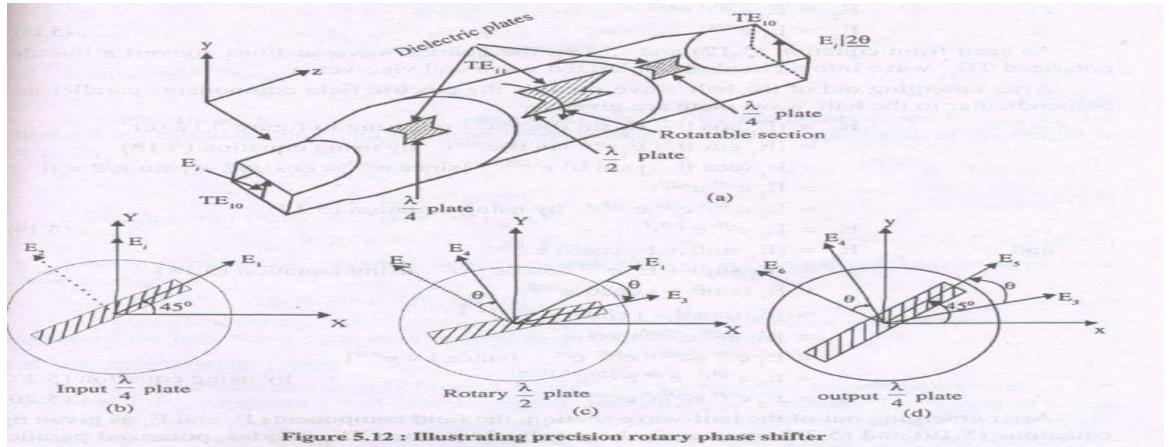


Figure 5.12 : Illustrating precision rotary phase shifter

When TEIO mode is propagated through the input rectangular waveguide of the rectangular to circular transition, then it is converted into TELL in the circular waveguide section. Let E_i be the maximum electric field strength of this mode which is resolved into components, E_1 parallel to the plate and E_2 perpendicular to E_1 as shown in figure 5.12 (b). After propagation through the plate these components are given by

$$\begin{aligned} E_1 &= (E_i \cos 45^\circ) e^{-j\beta_1 l} = E_0 e^{-j\beta_1 l} \\ E_2 &= (E_i \sin 45^\circ) e^{-j\beta_2 l} = E_0 e^{-j\beta_2 l} \end{aligned}$$

and

$$E_0 = \frac{E_i}{\sqrt{2}}$$

Where

The length l is adjusted such that these two components E_1 and E_2 have equal amplitude but differing in phase by $= 90^\circ$.

$$\begin{aligned} E_1 &= E_0 e^{-j\beta_1 l} \\ E_2 &= E_0 e^{-j(\beta_1 l - 90^\circ)} = E_0 e^{-j(\beta_1 l - \frac{\pi}{2})} \\ \therefore E_2 &= E_0 e^{-j\beta_1 l} e^{j\pi/2} \\ \therefore E_2 &= E_1 e^{j\pi/2} \end{aligned}$$

The quarter wave sections convert a linearly polarized TELL wave into a circularly polarized wave and vice-versa. After emerging out of the half-wave section, the electric field components parallel and perpendicular to the half-wave plate are given by

$$\begin{aligned}
E_3 &= (E_1 \cos \theta - E_2 \sin \theta) e^{-j2\beta_1 l} \quad \text{referring to figure 5.12 (c)} \\
&= (E_1 \cos \theta - E_1 e^{j\pi/2} \sin \theta) e^{-j2\beta_1 l} \quad \text{by using equation (5.18)} \\
&= E_1 (\cos \theta - j \sin \theta) e^{-j2\beta_1 l} \quad [\text{since } e^{j\pi/2} = \cos \pi/2 + j \sin \pi/2 = j] \\
&= E_1 e^{-j\theta} e^{-j2\beta_1 l} \\
&= E_0 e^{-j\beta_1 l} e^{-j\theta} e^{-j2\beta_1 l} \quad \text{by using equation (5.17)} \\
\therefore E_3 &= E_0 e^{-j\theta} e^{-j3\beta_1 l} \quad \dots (5.19)
\end{aligned}$$

and

$$\begin{aligned}
E_4 &= (E_1 \sin \theta + E_2 \cos \theta) e^{-j2\beta_2 l} \\
&= (E_1 \sin \theta + E_1 e^{j\pi/2} \cos \theta) e^{-j2\beta_2 l} \quad \text{using equation (5.18)} \\
&= E_1 (\sin \theta + j \cos \theta) e^{-j2\beta_2 l} \\
&= j E_1 (\cos \theta - j \sin \theta) e^{-j2(\beta_1 l - \frac{\pi}{2})} \\
&= j E_1 e^{-j\theta} e^{-j2\pi\beta_1 l} e^{j\pi} \\
&= E_1 e^{-j\theta} e^{-j2\beta_1 l} e^{j\pi/2} e^{j\pi} \quad [\text{since } j = e^{j\pi/2}] \\
&= E_0 e^{-j\beta_1 l} e^{-j\theta} e^{-j2\beta_1 l} e^{j3\pi/2} \quad \text{by using equation (5.17)} \\
\therefore E_4 &= E_0 e^{-j\theta} e^{-j3\beta_1 l} e^{j3\pi/2} \quad \dots (5.20)
\end{aligned}$$

After emerging out of the half-wave section, the field components E_3 and E_4 as given by equations (5.19) and (5.20), may again be resolved into two TE modes, polarized parallel and perpendicular to the output quarterwave plate. At the output end of this quarterwave plate, the field components parallel and perpendicular to the quarter wave plate, by referring to figure 5.12 (d), can be expressed as

$$\begin{aligned}
E_5 &= (E_3 \cos \theta + E_4 \sin \theta) e^{-j\beta_1 l} \\
&= (E_0 e^{-j\theta} e^{-j3\beta_1 l} \cos \theta + E_0 e^{-j\theta} e^{-j3\beta_1 l} e^{j3\pi/2} \sin \theta) e^{-j\beta_1 l}
\end{aligned}$$

$$\begin{aligned}
&= E_0 (\cos\theta + e^{j3\pi/2} \sin\theta) e^{-j\theta} e^{-j3\beta_1 l} e^{-j\beta_1 l} \\
&= E_0 (\cos\theta - j \sin\theta) e^{-j\theta} e^{-j4\beta_1 l} \\
\therefore E_5 &= E_0 e^{-j\theta} e^{-j\theta} e^{-j4\beta_1 l} \\
\therefore E_5 &= E_0 e^{-j2\theta} e^{-j4\beta_1 l} \quad \dots (5.21) \\
\text{and } E_6 &= (E_4 \cos\theta - E_3 \sin\theta) e^{-j\beta_2 l} \\
\therefore E_6 &= (E_0 e^{-j\theta} e^{-j3\beta_1 l} e^{j3\pi/2} \cos\theta - E_0 e^{-j\theta} e^{-j3\beta_1 l} \sin\theta) e^{-j\beta_2 l} \text{ by using equations (5.19) and (5.20)} \\
\therefore E_6 &= E_0 (e^{j3\pi/2} \cos\theta - \sin\theta) e^{-j\theta} e^{-j3\beta_1 l} e^{-j(\beta_1 l - \frac{\pi}{2})} \\
&= E_0 (-j \cos\theta - \sin\theta) e^{-j\theta} e^{-j3\beta_1 l} e^{-j\beta_1 l} e^{j\pi/2} \\
&= E_0 (-j) (\cos\theta - j \sin\theta) e^{-j\theta} e^{-j4\beta_1 l} e^{j\pi/2} \\
&= E_0 e^{j3\pi/2} e^{-j\theta} e^{-j\theta} e^{-j4\beta_1 l} e^{j\pi/2} \\
&= E_0 e^{-j2\theta} e^{-j4\beta_1 l} e^{j2\pi} \\
\text{since } e^{j2\pi} &= 1, \text{ we get} \\
E_6 &= E_0 e^{-j2\theta} e^{-j4\beta_1 l} \quad \dots (5.22)
\end{aligned}$$

Comparison of equation (5.21) and (5.22) yields that the components E_5 and E_6 are identical in both magnitude and phase and the resultant electric field strength at the output is given by

$$\begin{aligned}
E_{\text{out}} &= \sqrt{(E_5)^2 + (E_6)^2} \\
&= \sqrt{2} E_0 e^{-j2\theta} e^{-j4\beta_1 l}
\end{aligned}$$